Problem 1 Consider the system

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

Sketch the phase diagram of the system with orientations.

Problem 2 Consider the dynamical system

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ a & b \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}, \quad a \neq 0. \tag{1}$$

For which a and $b \in \mathbb{R}$ is $(\tilde{x}(t), \tilde{y}(t)) = (x(-t), -y(-t))$ a solution to (1) whenever $(x(t), y(t))$ solves (1)? Classify the equilibrium point in this case and determine if it is (Liapunov) stable, asymptotically stable or unstable.

Problem 3 Consider the system

$$\begin{aligned} \dot{x} &= \frac{7}{3}y \\ \dot{y} &= (x - 2e^{y})(x + e^{-y}). \end{aligned}$$

a) Find and classify all equilibrium points of the system.

b) Sketch the phase diagram of the system with orientations.

Problem 4 Consider the system

$$\begin{aligned} \dot{x} &= 2(x-1)g(x,y) + y \\ \dot{y} &= -(x-1) + 2yg(x,y), \end{aligned}$$

where $g(x,y) = 4 - x^2 - y^2$.

a) Determine if $(1,0)$ is a (Liapunov) stable, asymptotically stable or unstable equilibrium point of the system.

b) Determine whether or not this system has non-constant periodic solutions.

Problem 5 Consider the system

$$\dot{x} = x(y - x^2)$$
$$\dot{y} = (x - y)(x + y^2).$$

a) Compute the index of the origin.

b) Compute the index of the curve $C = \{(x, y) \in \mathbb{R}^2 \mid (x - \frac{1}{2})^2 + (y - \frac{1}{2})^2 = 1\}$.

Problem 6 Compute, with the variation of parameter method, the solution to the initial value problem

$$\dot{x} = -2x + (1 + 2t^2)e^{(1-t)^2}, \quad x(1) = 1.$$

Problem 7 Consider the system

$$\dot{x} = 1$$
$$\dot{y} = 3.$$

Define what it means for a solution to be (Liapunov) stable for $t \geq t_0$. Determine whether or not the solutions to the above system are (Liapunov) stable for $t \geq 0$.