Problem 1

a. Draw a phase portrait with orientations for the system

$$\dot{x} = -y,$$
$$\dot{y} = x - 2y.$$

b. Rewrite the system in polar coordinates. Use the rewritten system to verify the main features of your phase portrait. Explain briefly.

Problem 2 Show that the system

$$\dot{x} = 5x + y + x^2y - xy^2,$$
$$\dot{y} = x - 3y - x^2y - xy^2$$

has no non-constant periodic solution in the region $x^2 + y^2 < 2$.

Problem 3 Show that the origin is an asymptotically stable equilibrium for the system

$$\dot{x} = -2x - 2y + 2yz,$$
$$\dot{y} = x - y - xz - xz^2,$$
$$\dot{z} = 3xyz - z^3.$$

Problem 4

a. Show that the system

$$\dot{x} = x^2y + 2y^3,$$
$$\dot{y} = -2x^3 - xy^2$$

is Hamiltonian, and determine a Hamiltonian function for it.

b. Does the system

$$\dot{x} = x^2y + 2y^3 + (1 - x^2 - y^2)x,$$
$$\dot{y} = -2x^3 - xy^2 + (1 - x^2 - y^2)y$$

have a non-constant periodic solution?

Hint: Compute $\dot{H} = dH/dt$, where H is the Hamiltonian from the question above.

Problem 5 Consider the dynamical system

$$\dot{x} = y^2 - x^4,$$
$$\dot{y} = x^2 - y^4.$$

a. Show that the reflection through either of the diagonal lines given by $x = y$ or $x = -y$, respectively, maps each phase path of the system to another phase path. For each of the two reflections, indicate whether it preserves or reverses the direction of the phase paths.

b. Find and classify the equilibrium points of the system. For each one, indicate if it is stable, asymptotically stable, or unstable.

c. Compute the index of each equilibrium point.

d. Compute eigenvalues and eigenvectors of the linearization at each equilibrium point.

e. Sketch the phase diagram with orientations.