Norwegian University of Science and Technology
Department of Mathematical Sciences

For questions during the exam contact:
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EXAM in TMA4165 Differential equations and dynamical systems

English
Thursday 5th June 2008
09:00 – 13:00

Permitted aids (code D): Simple calculator (HP 30S)

Justify all answers.

Grading done by: 26th June 2008

Problem 1 Show that the origin is an asymptotically stable equilibrium point for the system

$$\begin{aligned}\dot{x} &= -2x + y - 2yz\\ \dot{y} &= -2x - y + 2xz - xz^2\\ \dot{z} &= 3xyz - z^5 + 6xy\end{aligned}$$

Problem 2 Consider the system

$$\begin{aligned}\dot{x} &= x + y^2\\ \dot{y} &= -y + y^3\end{aligned}$$

a. Find and classify the equilibrium points for the system, and sketch a phase diagram.

b. Show that the system has no non-constant periodic solutions.

Problem 3 (Counts as two points.) Show by sketches how the phase diagram (with orientation) for the system

$$\begin{aligned}\dot{x} &= -x - \mu x + y\\ \dot{y} &= -x - y + \mu y\end{aligned}$$

varies with the real number μ.

The problems continue on the back of the sheet.

Problem 4

a. Show that the system

$$\dot{x} = 2y(1 + e^{-x})$$
$$\dot{y} = e^{-x}(y^2 - 1)$$

is Hamiltonian, and find a Hamiltonian (function) for the system.

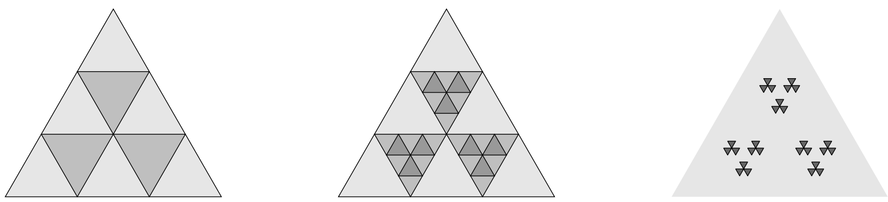
b. Find an equation for the two phase curves which separate those phase curves that cross the x axis from those that do not. Sketch the phase diagram for the system.

Problem 5 For all equilibrium points of the system

$$\ddot{x} + \dot{x} + (\mu^2 - x)(\mu - x^2) = 0$$

for all real values of μ, decide when these are stable and when they are unstable. Sketch the result in the (μ, x) plane.

Problem 6 The picture on the left shows a triangle divided into nine smaller triangles of equal size.



A fractal set is formed by removing everything except three of the smaller triangles, so that we are left with the darker part of the picture on the left. Then we remove everything from each of the small triangles except three subtriangles of each, just like in the big triangle, so that we are left with the still darker part of the middle picture. This process is repeated infinitely often, and the fractal is the set we are left with in the end.

Compute the fractal dimension of this set. Also compute the fractal dimension of the set that is formed by keeping instead the six dark triangles as shown in the picture below, and similarly in each step thereafter.