Problem 1 Consider the system

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} -5 & -2 \\ 4 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

Sketch the phase diagram of the system with orientations.

Problem 2 Consider the dynamical system

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}, \quad ad - bc \neq 0.$$

For which a, b, c, $d \in \mathbb{R}$ is the corresponding phase diagram with orientations symmetric with respect to the y-axis?

Problem 3 Consider the following two dynamical systems (in polar coordinates)

$$\dot{r} = r(r-\mu)(r-2), \quad \dot{\theta} = 1, \tag{1}$$

and

$$\dot{r} = r(r-\mu)^2(r-2), \quad \dot{\theta} = 1. \tag{2}$$

Assume that $\mu \geq 1$. Determine which of the two systems has a bifurcation point.

Problem 4 Given the dynamical system

$$\begin{aligned} \dot{x} &= x + y - 2 \\ \dot{y} &= x^2 + 2x - y - 2. \end{aligned}$$

a) Find and classify all equilibrium points of the system. Determine for each equilibrium point, if it is (Liapunov) stable, asymptotically stable or unstable.

b) Sketch the phase diagram of the system with orientations.

Problem 5 Consider the system

$$\begin{aligned} \dot{x} &= x^3 + xy^2 \\ \dot{y} &= y^3 - x^2 y. \end{aligned}$$

Determine for which pairs $(x(0), y(0))$ there exists a global solution.

Problem 6 Consider the system

$$\dot{x} = (-1 + \frac{e^{-t}}{1+t^2})x + (3 + te^{-t^2})y$$
$$\dot{y} = \frac{-2-3t^2}{1+t^2}x - y.$$

Determine if the origin is a (Liapunov) stable, asymptotically stable or unstable equilibrium point.

Problem 7 Consider the differential equation

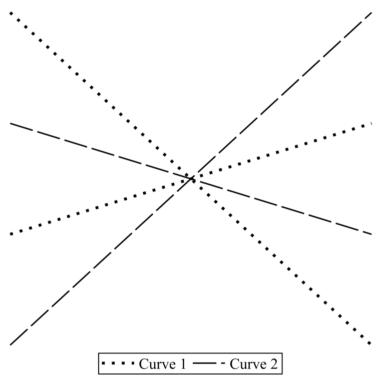
$$\dot{x} = x - \frac{1}{1+\varepsilon^2}x^2, \quad x(0) = \frac{1}{2}, \quad \varepsilon \in \mathbb{R}. \tag{3}$$

a) Show, with the help of a Grönwall estimate, that the initial value problem cannot have more than one solution.

b) Denote by x^0 and x^ε the solution to (3) with $\varepsilon = 0$ and $\varepsilon \neq 0$, respectively. Show that there exists a function $K(t)$ independent of ε such that

$$|x^0(t) - x^\varepsilon(t)| \leq K(t)\varepsilon^2.$$

Problem 8 Complete the following phase diagram close to the equilibrium point, so that the equilibrium point has index 2.



Curve 1 corresponds to $\{(x,y) \in \mathbb{R}^2 \mid \dot{y} = 0\}$.
Curve 2 corresponds to $\{(x,y) \in \mathbb{R}^2 \mid \dot{x} = 0\}$.