Homework Week 07

1. From the textbook: 19.1 (see definition 19.1.1), 19.4, 22.1, 22.2, 22.3,

2. (Version of the uncertainty principle.) Consider the operator L which acts on functions in $\mathcal{S}$ by the formula

$$Lf(x) = -f''(x) + x^2 f(x).$$

Let $\langle \cdot, \cdot \rangle$ stay for the usual inner product:

$$\langle f, g \rangle = \int_{-\infty}^{\infty} f(t)\overline{g(t)}dt, \quad f, g \in \mathcal{S}.$$

Prove that

$$\langle Lf, f \rangle \geq \langle f, f \rangle \text{ for all } f \in \mathcal{S}.$$

Comment. L is called Hermite operator. The later inequality can be written as $L \geq I$, where I, here I is the unit operator.