Homework week 12

1. Lesson 41. Exercises 41.1 -41.3

2. Let $g(t)$ satisfy $|g(t)| \leq C(1+|t|)^{-2}$ and $\|g\|_2 = 1$. Then the short time Fourier transform $V_g f(t,\xi)$ may be defined for functions $f \in L^\infty$. Assume that f is a unimodular function: $f(t) = \exp(i\phi(t))$, ϕ is real valued.

a. Prove that

$$\int_{-\infty}^{\infty} |V_g f(t,\xi)|^2 d\xi = 2\pi.$$

b. The quantity ϕ' plays the role of instanteneous frequency. Show that

$$\int_{-\infty}^{\infty} \xi |V_g f(t,\xi)|^2 d\xi = 2\pi \int_{-\infty}^{\infty} \phi'(t)|g(t-u)|^2 du.$$

Can you interpret this result ?