Lecture 01, 07.01.2014

- Periodic functions, reduction to the period 2π.
- Trigonometrical and exponential systems, trigonometrical and exponential polynomials
- Notion of Fourier series
- Orthogonality
- Formula for Fourier coefficients
- History digression: original motivation of Fourier
- Analogy with finite dimensional space
- The space $L^2(-\pi, \pi)$ (non rigorous modulo the Lebesgue measure)
- Energy interpretation of L^2 norm
- Formulation of the statement about expansion of functions in $L^2(-\pi, \pi)$.
- The exponential system spans the whole $L^2(-\pi, \pi)$ *(without proof)*
- Proof of expansion in $L^2(-\pi, \pi)$.
- Parseval identity

Topics to think about:

1. Make accurately construction of orthogonal projection and check all its properties

2. As we know we cannot write convergent expansions in polynomials, despite they generate the whole $L^2(-\pi, \pi)$. In which place the reasoning for Fourier series fail to work for polynomials?