Lecture 05 21.01.2014

- *Orthogonal expansions:*
  - Remind the general pattern
  - Definition: Hermite functions.
  - Structure of the Hermite functions, Hermite polynomials
  - Orthogonality of Hermite functions
  - *Exercise:* Calculate the norm of the Hermite functions
  - The Hermite functions span the whole $L^2(-\infty, \infty)$ *(without proof)*
  - The Sturm Liouville operator
  - Domain of definition (we take zero boundary conditions for simplicity)
  - Definition of spectra end eigenfunctions
  - Orthogonality of eigenfunctions
- *Isoperimetric problem*
  - Setting of the problem
  - Formulation in terms of the unknown functions $x(t)$, $y(t)$.
  - Parametrization by the arc length
  - Expression of the length of the curve in terms of the Fourier coefficients
  - Formula for the area
  - Expression of the area in terms of the Fourier coefficients
  - Inequality for the Fourier coefficients
  - Proof that the Fourier coefficients of order more than one vanish.
  - Proof that we obtain equation of a circle