NTNU – Trondheim
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Department of Mathematical Sciences

# Examination paper for **TMA4170 Fourier Analysis**

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**Examination time (from–to):** 09:00–13:00

**Permitted examination support material:** C: One A4-sized yellow sheet of paper stamped by the Department of Mathematical Sciences. A specific basic calculator is allowed.

**Language:** English

**Number of pages:** 2

**Number pages enclosed:** 0

**Checked by:**

Date Signature

**Notation** We use the Fourier transform in the form

$$\mathcal{F} : f \mapsto \hat{f}(\xi) = \int_{-\infty}^{\infty} f(t)e^{-2i\pi t\xi}dt.$$

**Problem 1** Let

$$\chi_{(-1,1)}(t) = \begin{cases} 0, & |t| > 1, \\ 1, & |t| < 1. \end{cases}$$

**a)** Find the Fourier transform of $\chi_{(-1,1)}(t)$.

**b)** Find the Fourier transform of $(1-t^2)\chi_{(-1,1)}(t)$.

**c)** Evaluate the integral

$$\int_0^{\infty} \frac{(\sin 2x)^2}{x^2}dx.$$

**Problem 2** Let $f(t)$ be a $2\pi$-periodic function defined by the relation $f(t) = \cos at$, $-\pi < t < \pi$, here $a$ is not an integer.

**a)** Find the Fourier series of $f$.

**b)** Prove the relation

$$\frac{1}{\sin \pi z} = \frac{1}{\pi z} + \sum_{k=1}^{\infty}(-1)^k \left[\frac{1}{\pi z - k\pi} + \frac{1}{\pi z + k\pi}\right].$$

*Remark.* You may (but not must) use the result of 2a.

**Problem 3** Find a solution to the equation

$$\left(\frac{d^2}{dx^2} - 9\right) u = \delta + e^{ix},$$

in the sense of distributions, here $\delta$ is the Dirac $\delta$-function.

**Problem 4** Let the distribution $T$ be defined by the formula $T = x\delta'$. Find $T'$.

**Problem 5** Consider the filter $\mathcal{A}(f) = g$, defined by the relation $g'' + \alpha g' + g = f$.

**a)** Determine the transmitting function $H$ and the impulse response $h = \mathcal{F}^{-1}H$.

**b)** Find whether this filter is stable and realizable for all values of $\alpha$. Explain your answer.

**Problem 6** Let $\phi$ and $\psi$ be the Haar scaling and wavelet functions respectively. Let $V_j$ and $W_j$ be the spaces generated by $\phi(2^j x - k)$, $k \in \mathbb{Z}$ and $\psi(2^j x - k)$, $k \in \mathbb{Z}$ respectively. Consider a function $f$ which vanishes outside $[0, 1)$ and on this interval it is delined by the relation

$$f(x) = \begin{cases} -1, & 0 \leq x < 1/4, \\ 4, & 1/4 \leq x < 1/2, \\ 2, & 1/2 \leq x < 3/4, \\ -3, & 3/4 \leq x < 1. \end{cases}$$

Express $f$ first in terms of the basis for $V_2$ and then decompose $f$ into its components parts in $W_1$, $W_0$, and $V_0$. In other words find the Haar wavelet decomposition of $f$. Explain your answers.