LINEAR PROGRAMMING IN

MATLAB OPTIMIZATION TOOLBOX

(may now be a little outdated!)

Basic function: **linprog**

Solves the general LP-problem

$$\min_x f'x,$$

$$Ax \le b$$

$$A_{eq.}x = b_{eq.}$$

$$lb \le x \le ub$$

where f, x, b, b_{eq}, lb, and ub are vectors and A, A_{eq} are matrices (may be entered as *sparse* matrices)

Syntax:

```
x       =       linprog( f, A, b, Aeq, beq)
x       =       linprog( f, A, b, Aeq, beq, lb, ub)
x       =       linprog( f, A, b, Aeq, beq, lb, ub, x0)
x       =       linprog( f, A, b, Aeq, beq, lb, ub, x0, options)

[x,fval]                             = linprog(...)
[x,fval,exitflag]                    = linprog(...)
[x,fval,exitflag,output]             = linprog(...)
[x,fval,exitflag,output,lambda]      = linprog(...)
```

Example: The Standard Form:

$$\min c'x,$$
$$Ax = b,$$
$$x \geq 0.$$

x = linprog(c,[ ],[ ],A,b,zeros(size(c)),[ ])

- Note the Matlab convention with *placeholders,* ”[ ]”

INPUT:

x0: Starting point. Used only for medium problems (*Nelder-Mead amoeba*).

Options:

Structure of parameters:

LargeScale:	'on'/’off’
Display:	'off'/'iter'/'final' (large scale problems)
MaxIter:	Max number of iterations
Simplex:	'on'/’off’ (‘on’ ignores x0)
TolFun:	Objective tolerance (large scale problems)

OUTPUT:

x,fval: Solution and objective

exitflag:

1	Iteration terminated OK
0	Number of iterations exceeded MaxIter
-2	No feasible point found
-3	Problem is unbounded
-4	NaN value encountered
-5	Both primal and dual are infeasible
-7	Search direction became too small

output: Structure of iteration information

iterations:	Number of iterations
algorithm:	Algorithm used
cgiterations:	The number of PCG iterations (large-scale algorithm only)
message:	Output message

lambda: Structure of Lagrange multipliers

ineqlin:	for linear inequalities $Ax \leq b$,
eqlin	for linear equalities $A_{eq}x = b_{eq}$,
lower	for lb,
upper	for ub.

ALGORITHMS:

Small/Medium scale: SIMPLEX-like including Phase 1

Large scale: Primal-dual inner method

EXAMPLES FROM THE DOCUMENTATION

A. Small Problem

Find ***x*** that minimizes

$$f(x) = -5x_1 - 4x_2 - 6x_3$$

subject to

$$\begin{aligned} x_1 - x_2 + x_3 &\le 20 \\ 3x_1 + 2x_2 + 4x_3 &\le 42 \\ 3x_1 + 2x_2 &\le 30 \\ 0 \le x_1, 0 \le x_2, 0 &\le x_3 \end{aligned}$$

First, enter the coefficients, then call **LINPROG**:

```
f = [-5  -4  -6]';
A =  [ 1 -1  1
        3  2  4
        3  2  0 ];
b = [20  42 30]';
lb = zeros(3,1);
[x,fval,exitflag,output,lambda] = … linprog(f,A,b,[],[],lb);
```

```
    x                =      [0  15  3]
    fval             =      -78.0
    output:
          iterations:   6
          algorithm:    'large-scale: interior point' (!)
          cgiterations: 0
          message:      'Optimization terminated.'

    lambda.ineqlin      =      [0  1.5  0.5]
    lambda.lower        =      [1  0  0]
```

For solution by the Simplex method:

```
f = [-5  -4  -6]';
A =  [ 1 -1  1
        3  2  4
        3  2  0 ];
b = [20  42 30]';
lb = zeros(3,1);

options = optimset('LargeScale','off','Simplex','on');
[x,fval,exitflag,output,lambda] = ...

linprog(f,A,b,[],[],lb,[],[],options);
```

(NB! If you forget enough placeholders, [ ], you get the error message ”LINPROG only accepts inputs of data type double”)

Now **output** gives:

```
  iterations:        3
  algorithm:         'medium scale: simplex'
  cgiterations:      []
  message:           'Optimization terminated.'
```

(same solution!)

B Medium Problem

This problem is stored as a Matlab MAT-file.

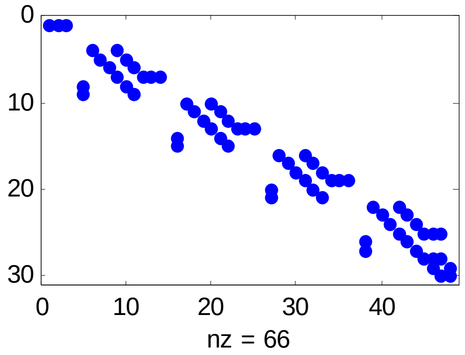
- 48 unknowns
- 30 inequality constraints
- 20 equality constraints
- $x \geq 0$

Entered into Matlab simply by

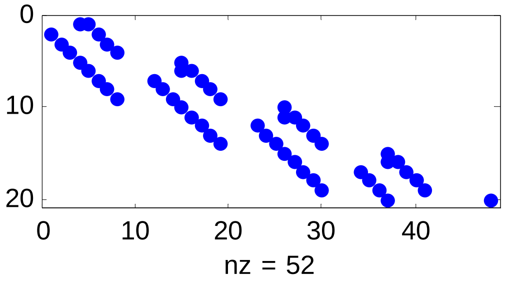
load sc50b

A	30x48	(sparse)
Aeq	20x48	(sparse)
b	30x1	
beq	20x1	
f	48x1	
lb	48x1	

Sparsity patterns:



A (inequalitites)



A_{eq} (equalities)

⇒

```
load sc50b

options = optimset('LargeScale','off','Simplex','on');

[x,fval,exitflag,output,lambda] =  ...
      linprog(f,A,b,Aeq,beq,lb,[],[],options);

x = [ 30  28 42 ... 102.4870]
```

In this particular case, only lambda.ineqlin(2) and lambda.ineqlin(3) were equal to 0, that is *only inequality 2 and 3 where non-active.*

max(lambda.lower)= 8.2808e-015 ⇒ $x_i > 0$ for $i = 1,\cdots,48$.

```
output =
      iterations:     43
      algorithm:      'medium scale: simplex'
      cgiterations:  []
      message:        'Optimization terminated.'
```

Problem run with large scale option:

```
options = optimset('LargeScale','on');
[x,fval,exitflag,output,lambda] = ...
                                   linprog(f,A,b,Aeq,beq,lb,[],[],options);
output =
      iterations:     8
      algorithm:      'large-scale: interior point'
      cgiterations:  0
      message:        'Optimization terminated.'
```

(Same solution!)

With display of results for each iteration:

options = optimset('LargeScale','on','Display','iter');

```
  Residuals:   Primal     Dual     Duality    Total
               Infeas    Infeas      Gap       Rel
               A*x-b    A'*y+z-f    x'*z      Error
  --------------------------------------------------
  Iter    0:  1.50e+03  2.19e+01  1.91e+04  1.00e+02
  Iter    1:  1.15e+02  3.18e-15  3.62e+03  9.90e-01
  Iter    2:  8.32e-13  1.96e-15  4.32e+02  9.48e-01
  Iter    3:  3.51e-12  1.87e-15  7.78e+01  6.88e-01
  Iter    4:  1.81e-11  3.50e-16  2.38e+01  2.69e-01
  Iter    5:  2.63e-10  1.23e-15  5.05e+00  6.89e-02
  Iter    6:  5.88e-11  2.72e-16  1.64e-01  2.34e-03
  Iter    7:  2.61e-12  2.59e-16  1.09e-05  1.55e-07
  Iter    8:  7.97e-14  5.67e-13  1.09e-11  3.82e-12
Optimization terminated.
```

FOR MORE INFO: Read documentation of **linprog!**