# UNCONSTRAINED OPTIMIZATION ALGORITHMS IN MATLAB / OPTIMIZATION TOOLBOX

For up-to-date information:
http://www.mathworks.se/help/techdoc/ref/helpbrowser.html

- Only the simplest algorithms are part of the basic distribution of MATLAB.

## FUNCTIONS IN THE STANDARD VERSION OF MATLAB

**fminbnd:**

Minimum of a function within an interval [a,b]

**fminsearch:**

Minimum of a function with *few* (2-6) variables (*Amoeba*)

# fminbnd

**SYNTAX:**

**x = fminbnd(fun,x1,x2)**
**x = fminbnd(fun,x1,x2,options)**
**x = fminbnd(fun,x1,x2,options,P1,P2,...) (obsolete)**
**[x,fval] = fminbnd(...)**
**[x,fval,exitflag] = fminbnd(...)**
**[x,fval,exitflag,output] = fminbnd(...)**

**fun:** m-file, *built-in function* or an *inline object*.
**[x1 x2]**: search interval
**options**: *Structure*[*)] of parameters. Set by the **optimset** function.
**P1, P2,** …: Parameters needed in and passed on to **fun**. **(obsolete)**

**ALLOWED OPTIONS:**

| | |
|---|---|
| **Display** | **off**=no output; **iter** =output at each iteration; **final** displays just the final output. |
| **MaxFunEvals** | Maximum number of function evaluations allowed. |
| **MaxIter** | Maximum number of iterations allowed. |
| **TolX** | Termination tolerance on x. |

*) Vector/array of “various things”

**EXAMPLE:**

**[x,fval,exitflag] =fminbnd('cos',3,4,optimset('TolX',1e-12,'Display','iter'))**

| Function-count | x | f(x) | Procedure |
|---|---|---|---|
| 1 | 3.38197 | -0.971249 | initial |
| 2 | 3.61803 | -0.888633 | golden |
| 3 | 3.23607 | -0.995541 | golden |
| 4 | 3.13571 | -0.999983 | parabolic |
| 5 | 3.1413 | -1 | parabolic |
| 6 | 3.14159 | -1 | parabolic |
| 7 | 3.14159 | -1 | parabolic |
| 8 | 3.14159 | -1 | parabolic |
| 9 | 3.14159 | -1 | parabolic |

*Optimization terminated successfully: The current x satisfies the termination criteria using OPTIONS.TolX of 1.000000e-012*

For a short version of output, use : **[x,fval,exitflag,output] = …**

# fminsearch

**SYNTAX**

```
x   =   fminsearch(fun,x0)
x   =   fminsearch(fun,x0,options)
[x,fval]                    = fminsearch(...)
[x,fval,exitflag]           = fminsearch(...)
[x,fval,exitflag,output]    = fminsearch(...)
```

**OPTIONS** structure fields:

As for **fminbnd** +

**Display** **simplex** (all corners)
**TolFun** Termination tolerance on the function value.

Note: The search strategy in Matlab also includes an ***outside contraction***

**EXAMPLE:**

**[x,fval] = fminsearch('banana', [-1.2, 1] , optimset('TolX',1e-3,'Display','iter'))**

| Iteration | Func.-count | min f(x) | Procedure |
|---|---|---|---|
| 1 | 3 | 6.361 | initial |
| 2 | 5 | 4.40994 | expand |
| 3 | 7 | 4.29678 | expand |
| 4 | 9 | 4.29678 | contract outside |
| 5 | 11 | 4.17987 | contract inside |
| 6 | 13 | 4.17987 | contract outside |
| 7 | 15 | 4.17987 | contract inside |
| 8 | 16 | 4.17987 | reflect |
| 9 | 18 | 4.07932 | expand |
| 10 | 19 | 4.07932 | reflect |
| ..... | | | |
| 63 | 119 | 2.0633e-007 | contract inside |
| 64 | 120 | 2.0633e-007 | reflect |
| 65 | 122 | 2.0633e-007 | contract inside |
| 66 | 124 | 2.0633e-007 | contract inside |
| 67 | 126 | 6.10245e-008 | contract inside |

# optimset

**(Type optimset on command line for full info about available options)**

## SYNTAX:

```
options = optimset( 'param1' , value1 , 'param2' , value2,...)
options = optimset
options = optimset(optimfun)
options = optimset(oldopts,'param1',value1,...)
options = optimset(oldopts,newopts)
```

## PARAMETERS:

## MATLAB AND OPTIMIZATION TOOLBOX:

| | |
|---|---|
| Display | [ off \| iter \| simplex\|{final} ] |
| MaxFunEvals | [ positive integer ] |
| MaxIter | [ positive integer ] |
| TolFun | [ positive scalar ] |
| TolX | [ positive scalar ] |

## MATLAB OPTIMIZATION TOOLBOX ONLY (*incomplete*):

| | |
|---|---|
| DerivativeCheck | [ on \| {off} ] |
| Diagnostics | [ on \| {off} ] |
| DiffMaxChange | [ positive scalar \| {1e-1} ] |
| DiffMinChange | [ positive scalar \| {1e-8} ] |
| GoalsExactAchieve | [ positive scalar integer \| {0} ] |
| GradConstr | [ on \| {off} ] |
| GradObj | [ on \| {off} ] |
| Hessian | [ on \| {off} ] |
| HessPattern | [ sparse matrix ] |
| HessUpdate | [ {bfgs} \| dfp \| gillmurray \| steepdesc ] |
| JacobPattern | [ sparse matrix ] |
| Jacobian | [ on \| {off} ] |
| LargeScale | [ {on} \| off ] |
| LevenbergMarquardt | [ on \| off ] |
| LineSearchType | [ cubicpoly \| {quadcubic} ] |
| MaxPCGIter | [ positive integer ] |
| MeritFunction | [ singleobj \| {multiobj} ] |
| MinAbsMax | [ positive scalar integer \| {0} ] |
| PrecondBandWidth | [ positive integer \| Inf ] |
| TolCon | [ positive scalar ] |
| TolPCG | [ positive scalar \| {0.1} ] |
| TypicalX | [ vector ] |

# ROUTINES IN MATLAB OPTIMIZATION TOOLBOX

**fminunc: Unconstrained minimization**

**Medium size:** BFGS Quasi-Newton (or Amoeba). Different strategies for line search.

**Large size:** Trust region methods. Different strategies for approximations to the Hessian matrix. Preconditioned Conjugate Gradient (PCG).

**lsqnonlin: Non-linear least square (includes *simple* interval constraints)**

**Medium size:** Gauss/Newton or Levenberg/Marquardt

**Large size:** Trust region methods or Gauss/Newton solved using PCG.

# fminunc

## Syntax

**x = fminunc(fun,x0)**
**x = fminunc(fun,x0,options)**

| | |
|---|---|
| **[x,fval]** | **= fminunc(...)** |
| **[x,fval,exitflag]** | **= fminunc(...)** |
| **[x,fval,exitflag,output]** | **= fminunc(...)** |
| **[x,fval,exitflag,output,grad]** | **= fminunc(...)** |
| **[x,fval,exitflag,output,grad,hessian]** | **= fminunc(...)** |

## Function, (gradient, (Hessian)):

```
function [f,g,H] = myfun(x)
    f = ...              % Compute the objective function value at x
    if nargout > 1       % fun called with two output arguments
      g = ...            % gradient of the function evaluated at x
      if nargout > 2     % fun called with three arguments
      H = ...            % Hessian evaluated at x
    end
```

**options = optimset('GradObj',’on’,'Hessian',’on’,...)**

## OPTIONS:

LargeScale

GradObj

### Medium-scale algorithm only:

DerivativeCheck
DiffMaxChange
DiffMinChange
LineSearchType

### Large-scale algorithm only:

HessPattern
MaxPCGIter
PrecondBandWidth
TolPCG
TypicalX

# Example: fminunc

```
function [f,g] = uncoex(x)
f = 3*(x(1)-1)^2 + 2*x(1)*x(2) + x(2)^2;
if nargout > 1
   g(1) = 6*(x(1)-1)+2*x(2);
   g(2) = 2*x(1)+2*x(2);
end

options = optimset('GradObj','on','Display','iter',);
x0 = [10,9];
[x,fval] = fminunc('uncoex',x0,options)
```

| Iteration | f(x) | Norm of step | First-order optimality *) | CG-iterations |
|---|---|---|---|---|
| 1 | 504 | 1 | 72 | 0 |
| 2 | 23.8973 | 10 | 12.5 | 1 |
| 3 | -1.5 | 4.34752 | 5.33e-015 | 1 |

***Optimization terminated successfully:***
***First-order optimality less than OPTIONS.TolFun, and no negative/zero curvature detected***

**x = [1.5000  -1.5000]**
**fval = -1.5000**

***) Infinity norm of the gradient.**

# lsqnonlin

## SYNTAX

```
x = lsqnonlin(fun,x0)
x = lsqnonlin(fun,x0,lb,ub)
x = lsqnonlin(fun,x0,lb,ub,options)

[x,resnorm]                                          = lsqnonlin(...)
[x,resnorm,residual]                                 = lsqnonlin(...)
[x,resnorm,residual,exitflag]                        = lsqnonlin(...)
[x,resnorm,residual,exitflag,output]                 = lsqnonlin(...)
[x,resnorm,residual,exitflag,output,lambda]          = lsqnonlin(...)
[x,resnorm,residual,exitflag,output,lambda,jacobian] = lsqnonlin(...)

function [F,J] = myfun(x)
F = ...              % objective function values at x
if nargout > 1       % two output arguments
   J = ...           % Jacobian of the function evaluated at x
end

options = optimset('Jacobian','on')
```

**Both the large-scale and medium-scale algorithms:**

Diagnostics
Display
Jacobian
MaxFunEvals
MaxIter
TolFun
TolX

**Medium-scale algorithm only:**

DerivativeChec
DiffMaxChange
DiffMinChange.
LevenbergMarquardt
LineSearchType

**Large-scale algorithm only:**

JacobPattern
MaxPCGIter
PrecondBandWidth
TolPCG.
TypicalX

# Example: lsqnonlin

```
function F = nonlsqfun(x)
k = 1:20;
F = 2 + 2*k-exp(k*x(1))-exp(k*x(2));
options = optimset('Display','iter');
x0 = [0.3 0.4];
[x,resnorm] = lsqnonlin('nonlsqfun',x0,[ ],[ ],options)
```

| Iter. | Func-count | f(x) | Norm of step | First-order optimality | CG-iterations |
|---|---|---|---|---|---|
| 1 | 4 | 2.048e+007 | 1 | 3.49e+008 | 0 |
| 2 | 7 | 5.80e+006 | 0.231124 | 1.15e+008 | 1 |
| 3 | 10 | 850620 | 1.04515 | 1.77e+007 | 1 |
| 4 | 13 | 850620 | 10 | 1.77e+007 | 1 |
| 5 | 16 | 850620 | 2.5 | 1.77e+007 | 0 |
| ---------- | | | | | |
| 30 | 91 | 1449.48 | 1.19209e-006 | 0.0177 | 0 |
| 31 | 94 | 1449.48 | 1.19209e-006 | 0.0177 | 1 |
| 32 | 97 | 1449.48 | 2.98023e-007 | 0.00378 | 0 |

Optimization terminated successfully:
Norm of the current step is less than OPTIONS.TolX
x = 0.1652 0.1652
resnorm = 1.4495e+003