TMA 4180 Optimeringsteori

Minimum Cost Network Flow Analysis Using LP

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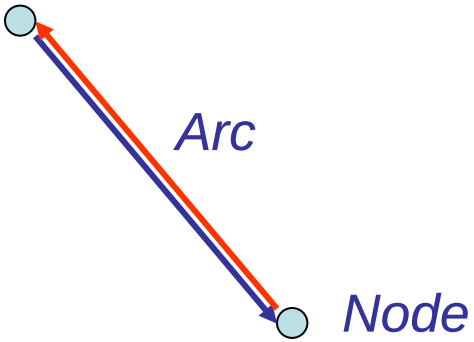
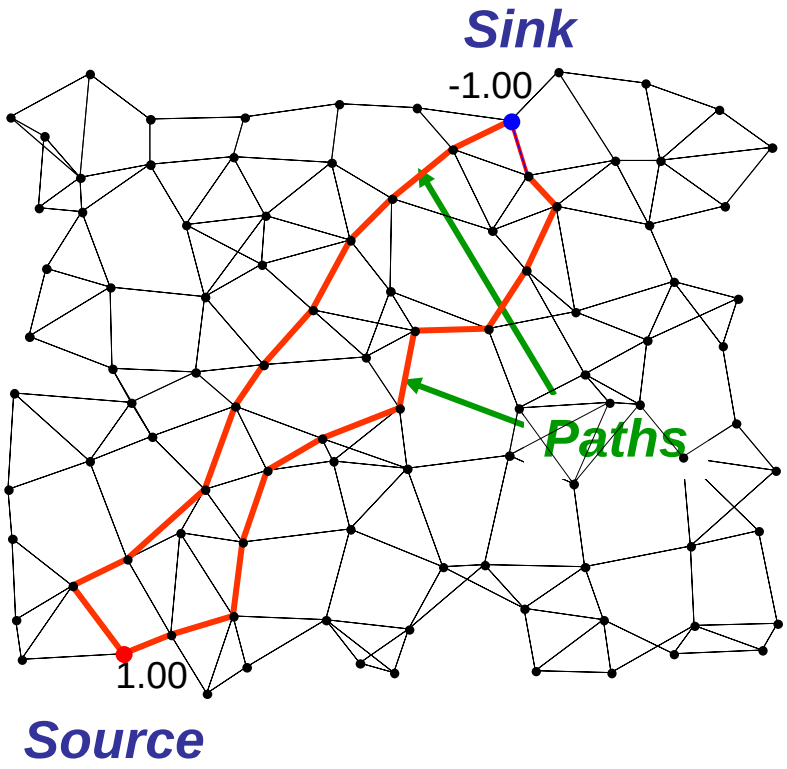


An arc is characterized by

- Prize pr. flow unit along arc
- Lower bound (for initiating transport)
- Upper bound (capacity)



Sources: (Production/providers)

- Capacity
- *Cost* pr. unit delivered to the network

Sinks (Consumers/receivers):

- Capacity
- *Income* to network from deliveries

Source: Production $b>0$.
Sink: Absorption, $b < 0$.

Variables $x = \{x_i\}, x_i \geq 0.$ (flow in the arcs)

NB! 2 variables for each arc: 2 *directions*

Node: $$\sum_{\text{inflow}} x_i = \sum_{\text{outflow}} x_i$$

Source/Sink: $$b_s = \sum_{\text{outflow}} x_i - \sum_{\text{inflow}} x_i$$

A *balanced* network: $$\sum_{\text{Sources/sinks}} b_s = 0$$

Price for delivery: $$f(x) = \sum_{\text{arcs}} c_i x_i = c'x$$

Cost for one unit along arc "*i*": $\{c_i\}$

Upper bound on capacity for arc "*i*": $\{ub_i\}$

Lower bound on capacity for arc "*i*": $\{lb_i\}$

The LP formulation:

$$\min_x c'x$$

$$\sum_{\text{outflow}} x_i - \sum_{\text{inflow}} x_i = b_n,\ n = 1,\ldots,Nodes,$$

$$lb \le x \le ub.$$

$$\min_x c'x$$

$$A_{eq}x = b_{eq}$$

$$lb \le x \le ub$$

The matrix is a *sparse* matrix with only -1, 0, and -1

Simsys_sparse

MATLAB CENTRAL

An open exchange for the MATLAB and Simulink user community

http://www.mathworks.com/matlabcentral/

Per Bergström
Luleå University of Technology

RANDOM NETWORK GENERATION

Prescribe:

- Numbers of sources and sinks
- Max number of arcs around one node
- Min number of arcs around one node
- Random upper bound
- Distribution of nodes
- Interactive network modification
- Random costs

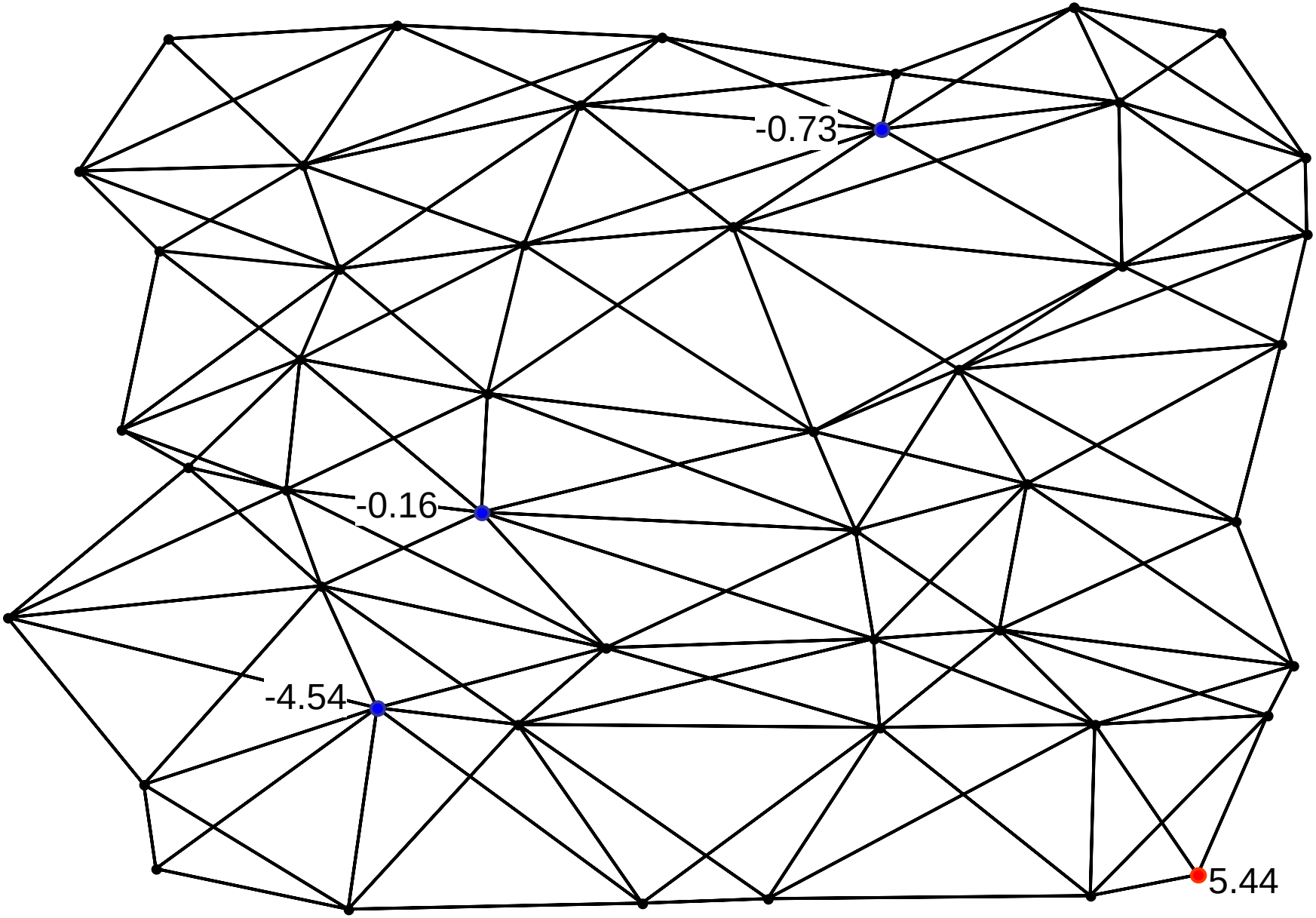
The algorithm provides:

- Number of nodes
- Upper bound of capacity
- A_{eq} matrix
- Balanced production/consumption at the sources and sinks

[Aeq,beq,lb,ub,c]=simsys_sparse(100);

Solution in Matlab: x = linprog(c,[],[],Aeq,beq,lb,ub)

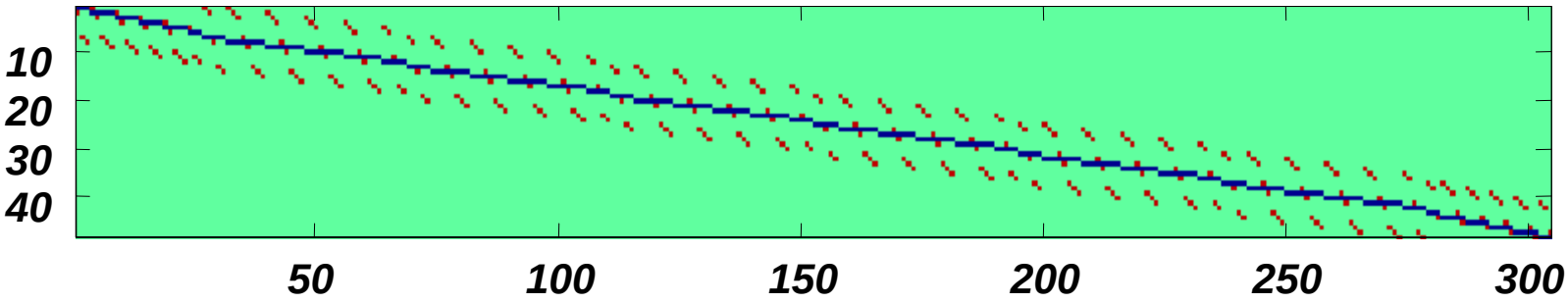
RANDOMLY GENERATED NETWORK

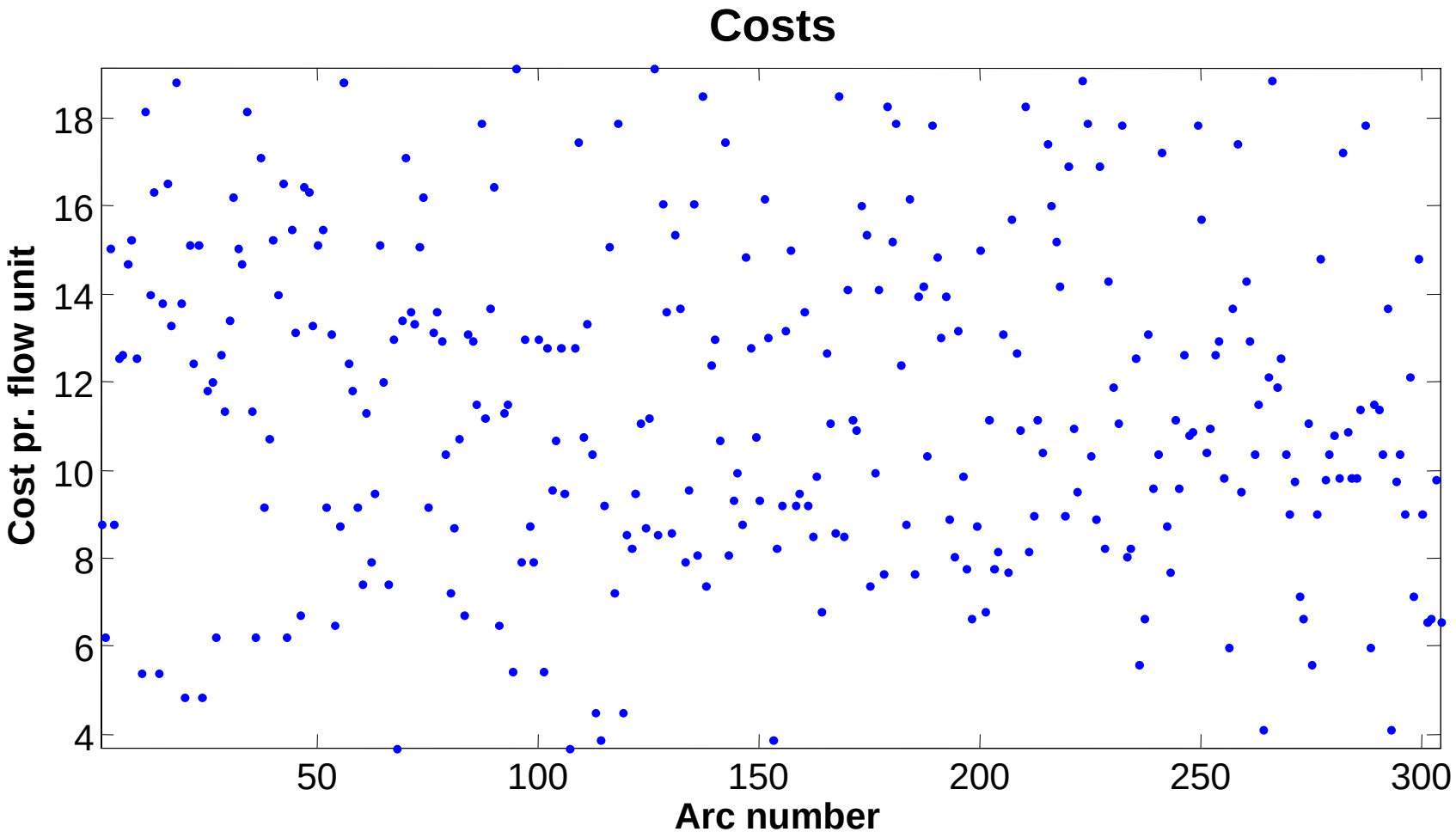


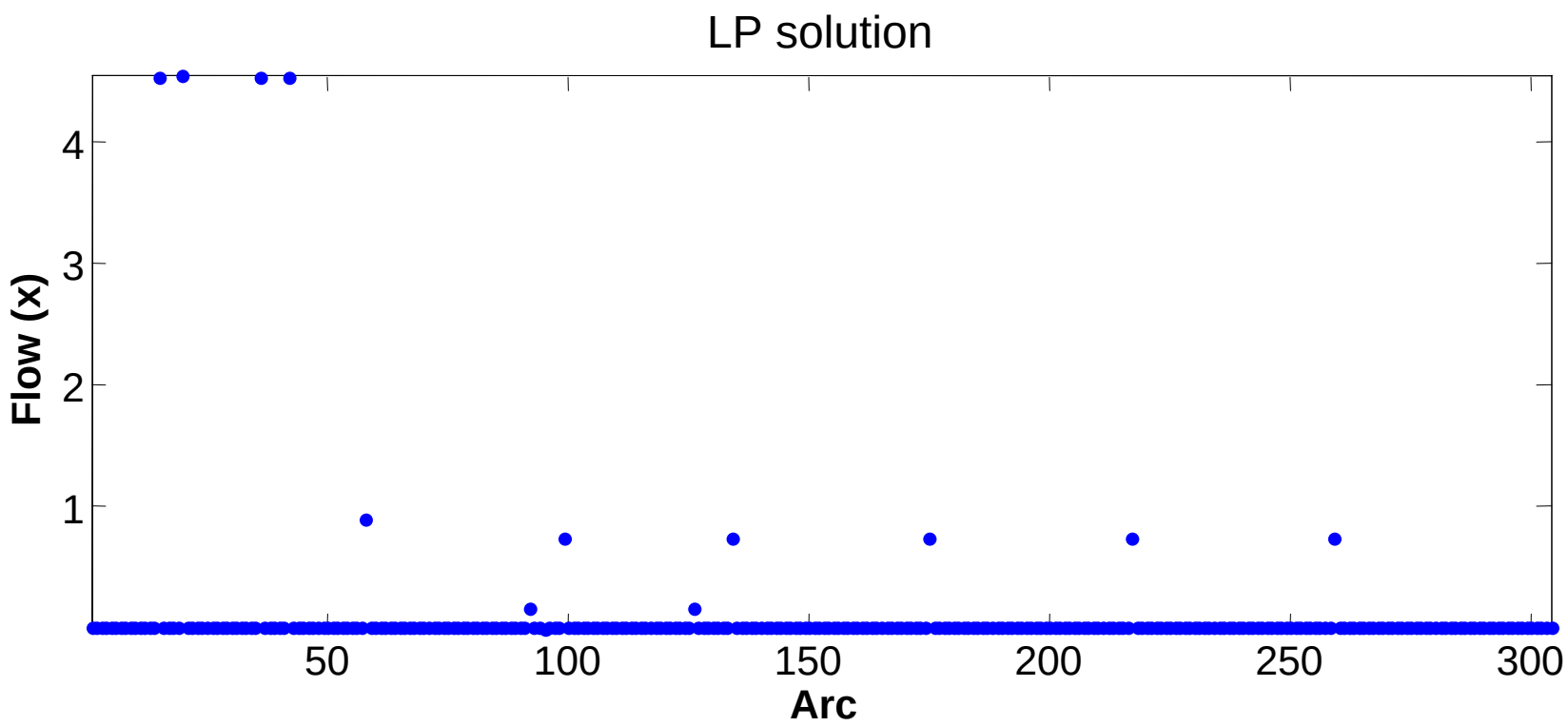
The LP-problem:

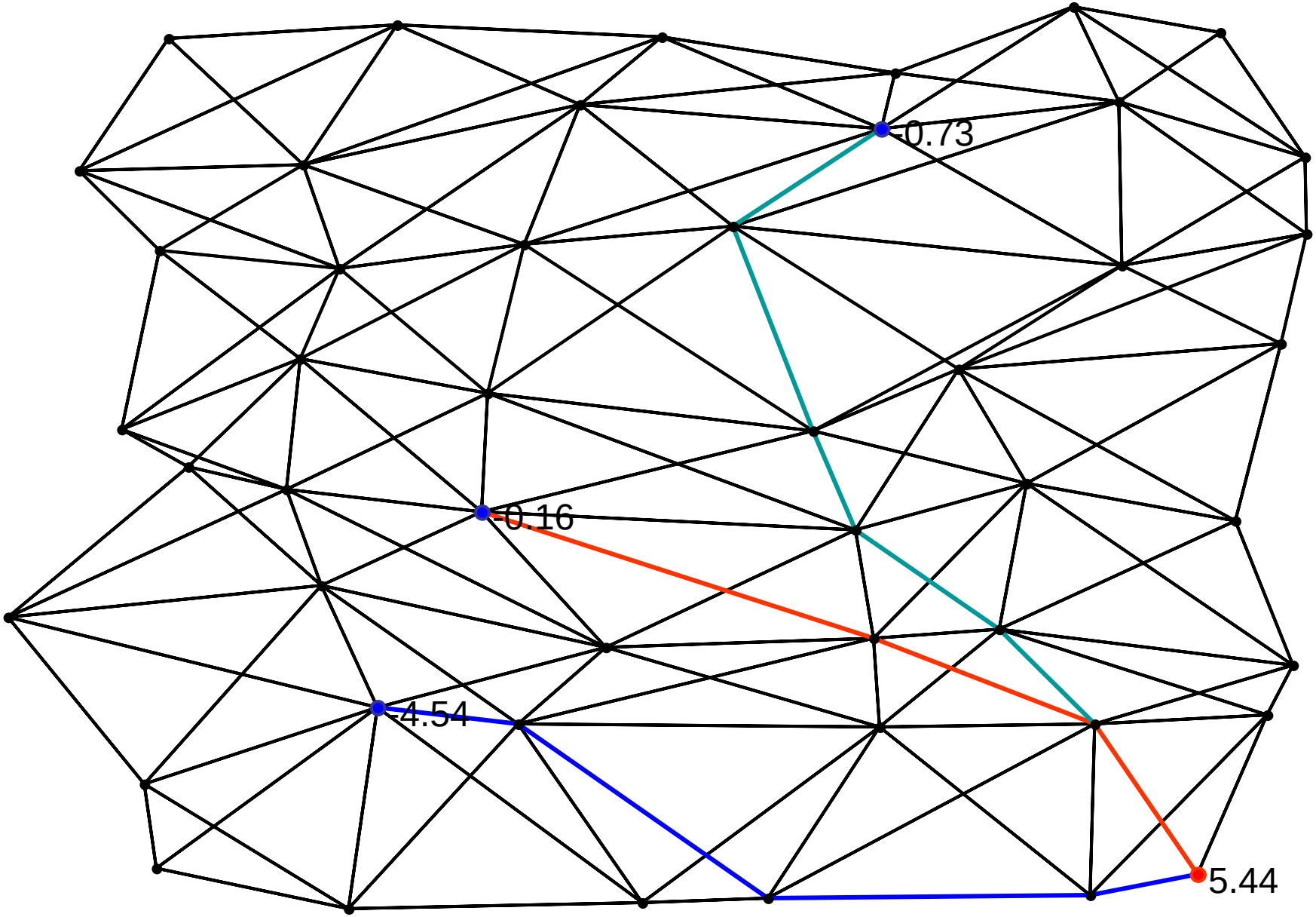
- Number of arcs: 304
- Lower bounds: 0
- Upper bounds: -
- Equality constraints: 48

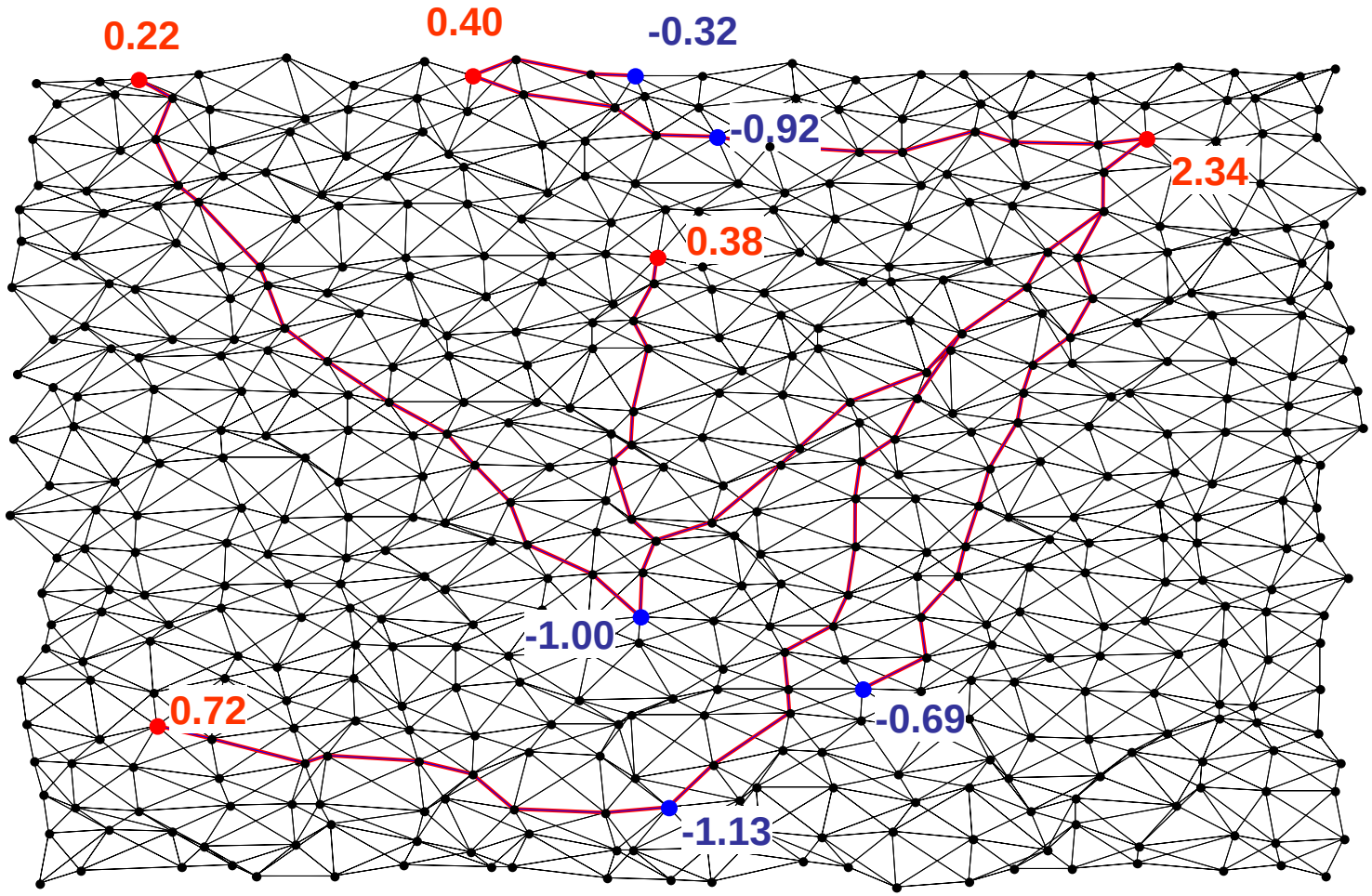
A_{eq}-matrix:




Costs
Cost pr. flow unit
Arc number
4
6
8
10
12
14
16
18
50
100
150
200
250
300

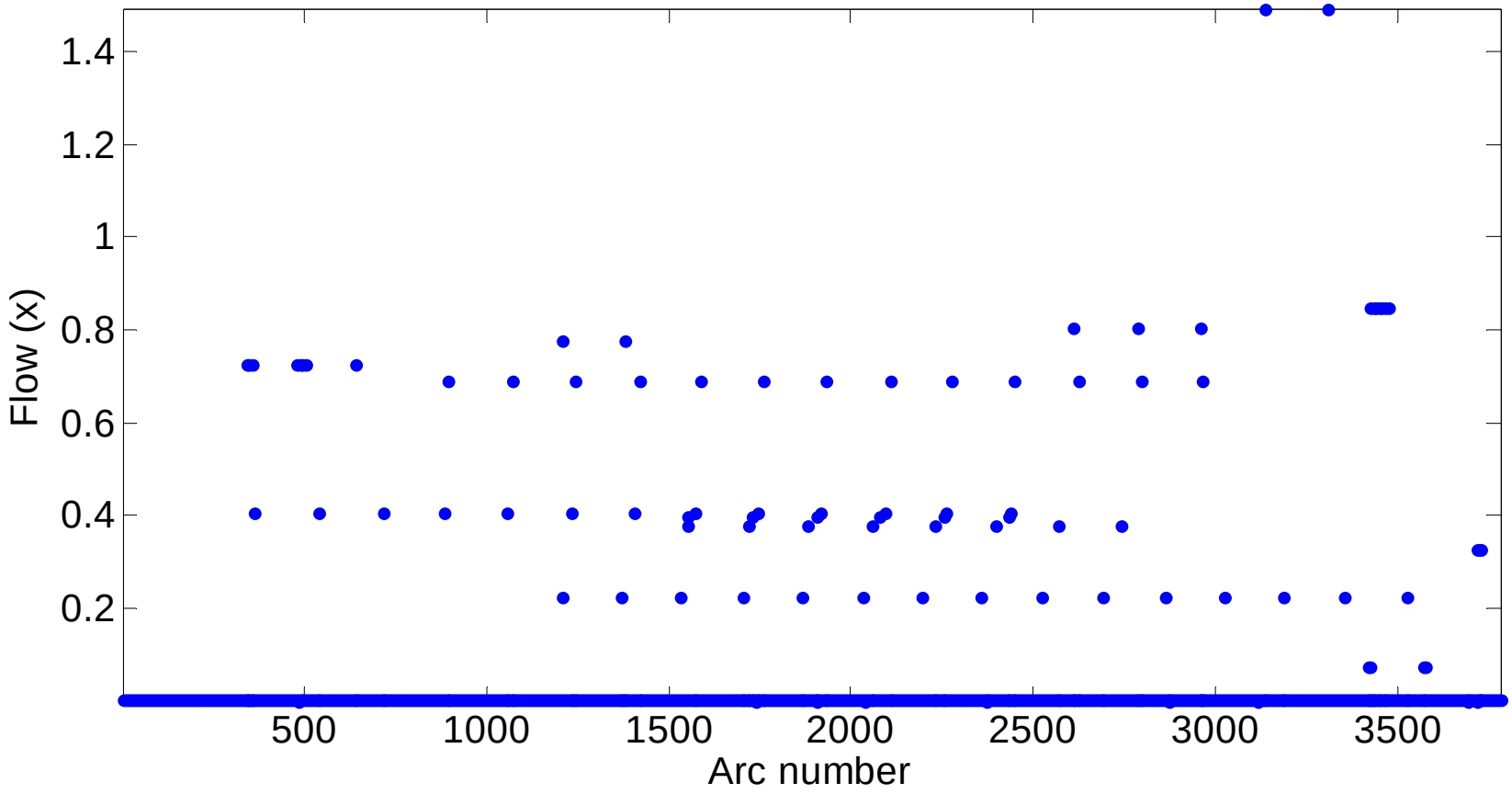

LP solution
Flow (x)
Arc
1
2
3
4
50
100
150
200
250
300


0.73
0.16
4.54
5.44



$$\dim\left(x\right)=3782$$

$$\dim\left(A_{eq}\right)=506\times3782$$


1.4
1.2
1
0.8
0.6
0.4
0.2
Flow (x)
500
1000
1500
2000
2500
3000
3500
Arc number

Practical Optimization: A Gentle Introduction

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http://www.sce.carleton.ca/faculty/chinneck/po.html

(*very* soft introduction ☺)