STEEPEST DESCENT FOR THE TEST PROBLEM

MATRIX GENERATION:

1. **First generate a random matrix:**

$$R = \mathrm{randn}(N, N)$$

2. **Form a positive definite matrix:**

$$A_\alpha = \left(R' R\right)^\alpha, 0 < \alpha < \infty$$

3. **Adjust the condition number:**

$$A_1: \quad \lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_N$$

$$A_\alpha: \quad \lambda_1^\alpha \leq \lambda_2^\alpha \leq \ldots \leq \lambda_N^\alpha$$

$$\kappa_\alpha = \kappa_1^\alpha, 0 < \alpha < \infty$$

ESSENTIAL ALGORITHM:

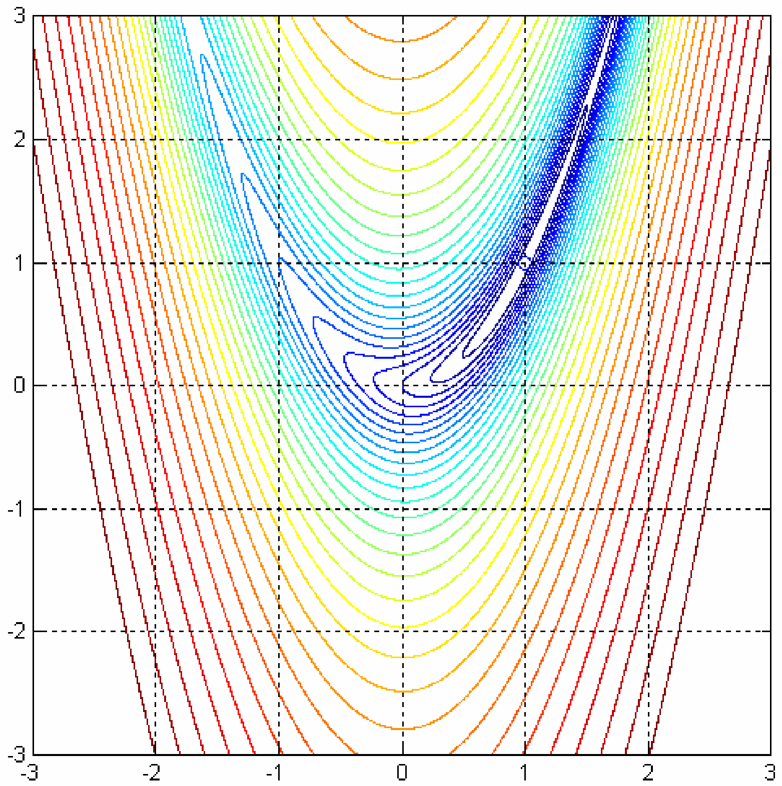
```
ndim = 100;
R    = randn(ndim);             % Random matrix
npot = 0.4;                     % Steers the eigenvalue dist.
A  = (R'*R)^npot;               % A matrix, >0.
lamb = eig(A);                  % Eigenvalues
kappa= max(lamb)/min(lamb) % Condition number
b    = rand(ndim,1);            % b-vector
xsol = -A\b;                    % Solution
%
for loop = 1 : Niterations
   Ag    = A*g;
   alpha = (g'*g) /(g'*Ag);
   x     = x - alpha*g;
   g     = g - alpha*Ag; %  g = b+A*x;
   err2(loop) = sqrt((x-xsol)'*(x-xsol))/Norm2;
   errA(loop) = sqrt((x-xsol)'*A*(x-xsol))/NormA;
end;
% Plots …
```

ROSENBROCK'S BANANA FUNCTION

Rosenbrock's function[1] :

$$f(x,y) = (1-x)^2 + a(y-x^2)^2, \left(x,y\right) \in \mathbf{R}^2, a > 0.$$

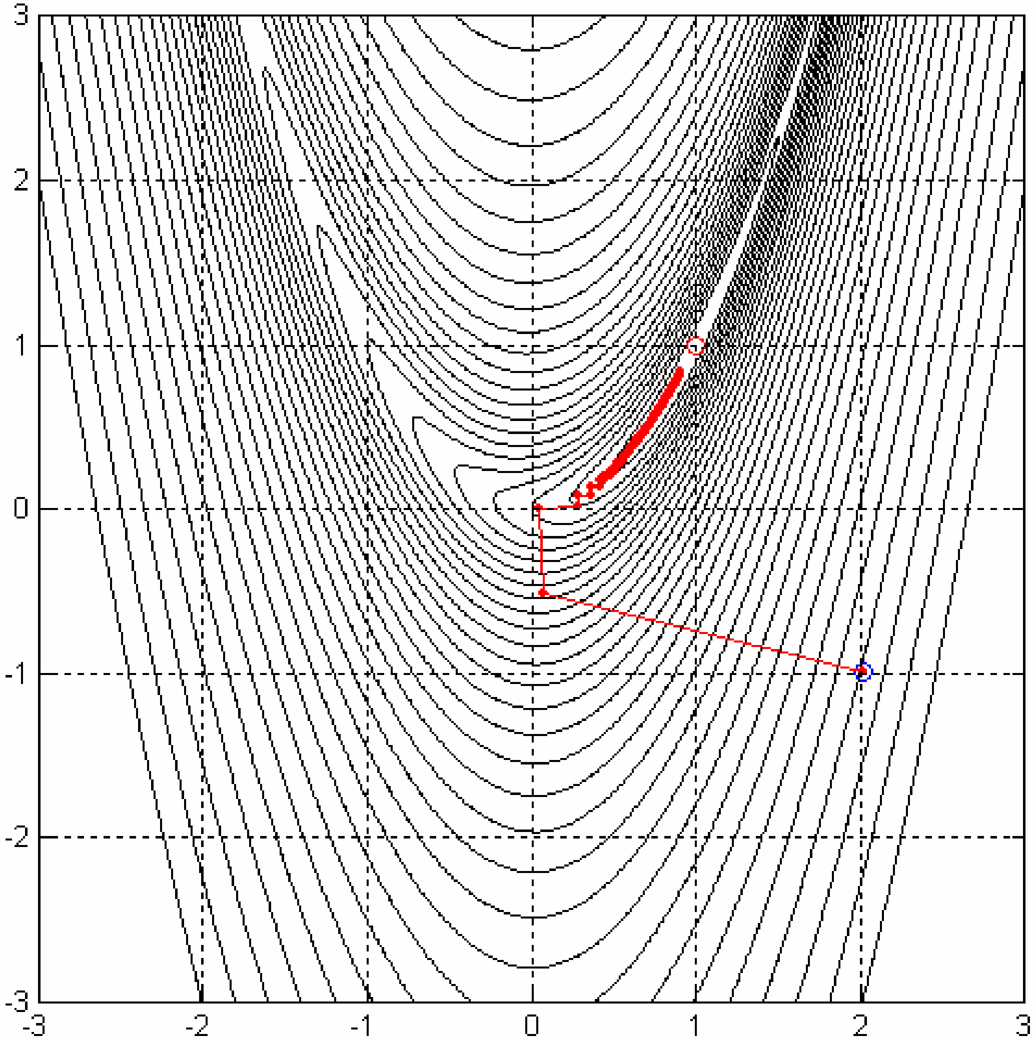
- **Unique minimum at (1,1)**
- **Convex at (1,1), but not always**



[1] Rosenbrock, H. H.: An Automatic Method for Finding the Greatest or Least Value of a Function, *Computer J.* **3** (1960) 175 – 184,

Steepest Descent Demo Program

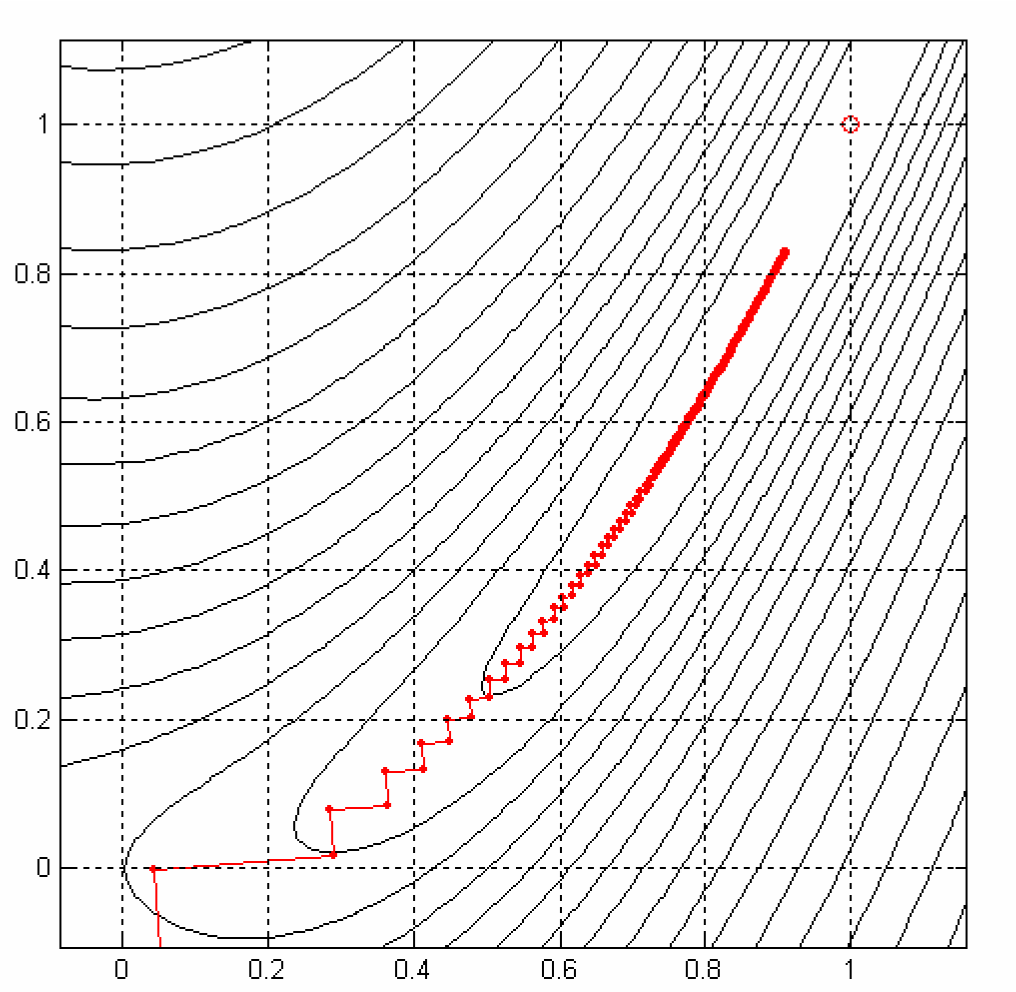
```
% Contour plot
[XG,YG] = meshgrid( -3 : 0.02 : 3 , -3 : 0.02 : 3 );
b =  Banana(XG,YG);
contour(XG,YG,log10(b+1.00001),-2 : 0.1 : 3 ,'k');
%
% Get starting point
X = ginput(1);
P = -BananaGradient(X);
%
% Iterate, Niter = 200
for loop = 1:Niter
            % Line search
            alfaopt = fminbnd  % 1D solver (details omitted)
            X = X + alfaopt*P;
            Xstore(:,loop) = X;
            P = -BananaGradient(X);
end;
plot( Xstore(1,:), Xstore(2,:) )
```



200 iterations –
far from the solution!

$$A = 20$$

Magnified view:



$$\nabla^2 f = \begin{bmatrix} (12ax^2 - 4ay + 2) & -4ax \\ -4ax & 2a \end{bmatrix}$$

At (1,1):

$$\nabla^2 f(1,1) = \begin{bmatrix} (8a+2) & -4a \\ -4a & 2a \end{bmatrix}$$

Eigenvalues:

$$\lambda_1 = 10a + O(1),$$

$$\lambda_2 = \frac{2}{5} + O\left(\frac{1}{a}\right).$$

$$\Rightarrow \nabla^2 f(1,1) > 0$$

Condition number:

$$\kappa = 25a + O(1)$$