TMA 4180 Optimeringsteori

Spring term 2013

Exercise 5

(Sorry for the layout! HEK)

Problem 1

N&W Exercise 5.4 (p. 133 in 2nd ed.). What important condition on the p-s is missing in the text? (Hint: Note that you may write $x_0 + P\sigma$, where $P = [p_0\ p_1\ \cdots p_{k-1}]$ and $\sigma = [\sigma_0\ \cdots \sigma_{k-1}]'$.)

Problem 2

In this problem we shall look at some statements you find in the textbooks about the CG-method. The following simple Matlab code for the CG-method of a quadratic problem is also stated in the note on the Web:

```
ndim = 100; R = randn(ndim); npot = .1;
A = (R'*R)^npot;
kappa= max(eig(A))/min(eig(A));
xsol = rand(ndim,1); b = A*xsol;
Norm2 = sqrt(xsol'*xsol); NormA = sqrt(xsol'*A*xsol);
x = zeros(size(b)); g = A*x-b ; p = -g;
for loop = 1:ndim
  Ap = A*p;
  alfa = -(p'*g)./(p'*Ap);
  x = x + alfa*p;
  g = g + alfa*Ap; % g = A*x-b;
  beta = (g'*Ap)./ (p'*Ap);
  p = -g + beta*p;
  err2(loop) = sqrt((x-xsol)'*(x-xsol))/Norm2;
  errA(loop) = sqrt((x-xsol)'*A*(x-xsol))/NormA;
end;
semilogy(1:ndim, err2,1:ndim,errA,'r');
legend( '2-norm' , 'A-norm' );
xlabel('Iteration number'); ylabel('Error')
Tittel = ['npot= ' num2str(npot) ' \kappa=',num2str(kappa)];
title(Tittel);
```

(a)

Implement and plot the error bound

$$\|x_k - x^*\|_A \leq 2\left(\frac{\sqrt{\kappa}-1}{\sqrt{\kappa}+1}\right)^k \|x_0 - x^*\|_A . \tag{1}$$

in the Matlab code above. How does this compare with the actual decrease of the error? N&W say: "*This bound often gives a large overestimate*". True?

(b)

Modify a well-conditioned matrix A so that it has m ($m = 3 - 6$) large eigenvalues by adding a random rank-m matrix LL',

$$A = \left(R'R\right)^{npot} + \mu LL', \ \mu \gg 1 \tag{2}$$

(L is $n \times m$ and consists just of m random column vectors). Test the performance of the CG-method in this case.

(Hint: Read about this in N&W p. 115 – 117 and the note on the web-page).

(c)

It is stated in the classic book by Luenberger (and also reproduced in the note) that in case (b) above, the CG should be restarted with a SD step every m-th step. Is this really necessary? (The SD step is obtained by setting $\beta = 0$ every m-th step),