Norwegian University of Science and Technology Department of Mathematical Sciences TMA4180 Optimization Theory Spring 2013

Exercise set 7

Tutorial: Thursday 07.03 16:15-17:00 in Kjl 4.

1 Problem 12.17 in N&W p. 353.

2 (Midterm Exam 2010)

Consider the following constrained optimization problem for  $(x_1, x_2) \in \mathbb{R}^2$ :

$$\min_{x\in\Omega}\left\{-4x_1 - x_2\right\},\tag{1}$$

where  $\Omega$  is defined in terms of the constraints

$$0 \le x_1 \le 2,\tag{2}$$

$$0 \le x_2,\tag{3}$$

$$x_2 \le 3 - x_1. \tag{4}$$

a) Reformulate the constraints into four constraints of the form

$$c_i(x) \ge 0, \ i = 1, \cdots, 4,$$
 (5)

and write down all KKT-equations and inequalities.

- **b)** Solve the problem graphically by making a sketch of  $\Omega$ .
- c) Identify the active and inactive constraints and the corresponding Lagrange multipliers at the solution.
- **3** Problem 12.21 in N&W, p. 354.

4 Consider the problem

$$\min(x_2 + x_3),$$
  
$$x \in \Omega = \left\{ x \; ; \; x_1 + x_2 + x_3 = 1, \; x_1^2 + x_2^2 + x_3^2 \ge 1 \right\}.$$

Note that the feasible domain is unbounded.

- **a)** Show that the only KKT-point for the problem is  $(-1, 2, 2)^{T}/3$ .
- **b)** Use the second order conditions to investigate whether this KKT-point really is a local minimum.