TMA4180 Optimization Theory
Spring 2013

Norwegian University of Science and Technology
Department of Mathematical Sciences

Exercise set 10

Tutorial: Thursday 25 16:15-17:00 in El 1 (NB! Change of room).

$\boxed{1}$ *Troutman*, Problem 1.2.

Hints: The transit time from $(0,0)$ to $(1,1)$ along a path $y(x)$, where $y(0) = 0$, $y(1) = 1$, is given by

$$T = \frac{1}{\sqrt{2g}} \int_0^1 \left(\frac{1 + y'(x)^2}{y(x)} \right)^{1/2} \mathrm{d}x,$$

and the problem is a technical exercise in estimating the value of this integral for various paths, $y = y(t)$.

For (c) you may use that

$$\int_0^{\pi/2} \frac{\mathrm{d}\theta}{(\sin\theta)^{1/2}} = \frac{1}{2}\pi^{3/2} \frac{\sqrt{2}}{\Gamma(3/4)^2} \approx 2.622.$$

Point (e) seems to be tricky, so try the not-so-obvious inequality

$$\sin\theta \geq \theta - \theta^2/\pi, \quad 0 \leq \theta \leq \pi/2.$$

Maybe you see a simpler way!

$\boxed{2}$ *Troutman*, Problem 2.5 (a), (c), (e).

Hints: In some of these and following problems you'll need to put $\mathrm{d}/\mathrm{d}\varepsilon$ inside the integral sign,

$$\frac{\mathrm{d}}{\mathrm{d}\varepsilon} \int_a^b h(x,\varepsilon)\,\mathrm{d}x = \int_a^b \frac{\partial h(x,\varepsilon)}{\partial\varepsilon}\,\mathrm{d}x. \tag{1}$$

Theorem A.13 in Troutman is a simple sufficient condition for this to be allowed: *Assume that* $[a,b]$ *is finite and* h *as well as* $\partial h/\partial\varepsilon$ *are continuous on* $[a,b] \times [\alpha,\beta]$. *Then* (1) *holds for all* $\varepsilon \in [\alpha,\beta]$.

All problems are most easily solved by applying the formula

$$\delta J(y; v) = \left.\frac{\partial}{\partial\varepsilon} J(y + \varepsilon v)\right|_{\varepsilon=0}.$$

$\boxed{3}$ *Troutman*, Problem 2.10 (a).

Hint: Use that $J(y + \varepsilon v) - J(y) = \varepsilon\delta J(y; v) + o(\varepsilon)$.

$\boxed{4}$ *Troutman*, Problem 2.12.

Hint: Consider also the convexity of this functional.

$\boxed{5}$ *Troutman*, Problem 3.6.

$\boxed{6}$ *Troutman*, Problem 3.7.

$\boxed{7}$ *Troutman*, Problem 3.28.

Hint: Verify that the given solution satisfies the Euler equation and the constraints.

$\boxed{8}$ *Troutman*, Problem 3.29.

Hint: The solution is a surprise!