Ex 201

b) WTS: $(fg)'(x) = f'(x)\,g(x) + f(x)\,g'(x)$.

First, suppose the product rule holds for $f(x)\cdot x^m$, for ~~some~~ all m. Then we have:

$$(fg)'(x) = \left(f(x)\,g(x)\right)' = \left(f(x)\cdot \sum_{i=0}^{\deg(g(x))} g_i x^i\right)' =$$

$$= \sum_{i=0}^{\deg(g(x))} g_i \left(f(x)\,x^i\right)' = \sum_{i=0} g_i \left(f'(x)\cdot x^i + f(x)\,(x^i)'\right) =$$

$$= \sum_{i=0} g_i f'(x)\, x^i + \sum_{i=0} g_i f(x)\,(x^i)' =$$

$$= f'(x) \sum_{i=0} g_i x^i + f(x)\left(\sum_{i=0} a_i x^i\right)' = f'(x)\,g(x) + f(x)\,g'(x).$$

Now we have reduced the problem to show that the rule holds for $f(x)\cdot x^m$. In the same way as above we can reduce this problem to the case $x^n\cdot x^m$. So we have

$$(x^n x^m)' = (x^{n+m})' = (n+m)\,x^{n+m-1}$$

$$(x^n)'\cdot x^m + x^n (x^m)' = n x^{n-1}\cdot x^m + x^n\cdot m x^{m-1} =$$

$$= n x^{m+n-1} + m x^{m+n-1} = (n+m)\,x^{m+n-1},$$

and we have equality and we are done! ∎