

Ex 201

c) WTS: $(f(x)^m)' = m(f(x))^{m-1} f'(x)$

Let $m=2$. Use induction:

$$\begin{aligned}(f(x)^2)' &= (f(x) \cdot f(x))' = f'(x)f(x) + f(x) \cdot f'(x) = \\ &= (f(x) + f(x)) \cdot f'(x) = 2f(x) \cdot f'(x).\end{aligned}$$

\Rightarrow True for $m=2$.

Suppose it is true for $m=i-1$.

$$\begin{aligned}(f(x)^i)' &= (f(x) \cdot f(x)^{i-1})' = f'(x) \cdot f(x)^{i-1} + f(x) \cdot (f(x)^{i-1})' \\ &= f'(x) \cdot f(x)^{i-1} + f(x) \cdot \underbrace{((i-1) f(x)^{i-2} \cdot f'(x))}_{\text{since true for } i-1}\end{aligned}$$

$$= f'(x) \cdot f(x)^{i-1} + (i-1) f(x)^{i-1} \cdot f'(x) =$$

$$= i f(x)^{i-1} \cdot f'(x), \text{ and hence it is true for } i, \text{ and we are done. } \square$$