Norwegian University of Science and Technology
Department of Mathematical Sciences

Contact during the exam:
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Exam in TMA4185 CODING THEORY
English
Wednesday June 7, 2006
Time: 09.00-14.00

Permitted aids:
Approved calculator
All printed or written aids

The grades are posted in week 26

Problem 1

What should be the minimal distance d for a code in order that it would be possible to correct 1 error and detect the situation when there are 2, 3 or 4 errors? Why?

Problem 2

Let $\mathcal{C}_1$ be the orthogonal to the binary Hamming [7,4]-code. Find the dimension k_1 and the minimal distance d_1 for $\mathcal{C}_1$. Write a generating matrix for $\mathcal{C}_1$.

Problem 3

Find a generating polynomial for the minimal binary cyclic code containing

$$\bar{v} = (000.011.111.010.000) \in \mathbb{F}_2^{15}.$$

Problem 4

Write the statement of the Gilbert-Varshamov theorem for codes over $\mathbb{F}_3$. Find the minimal value for n such that the G-V-theorem implies the existence of a code $C \subset \mathbb{F}_3^n$ with $k = 16$, $d = 4$.

Problem 5

The binary Golay code G_{24} was used and the received word

$$\bar{w} = 0x0.100..x0x.x00.010.110.100.110$$

happens to have 4 erasures. Find the codeword that was sent.

Problem 6

The normalized RS(32,5)- code was used with a generator $\alpha \in \mathbb{F}_{32}^*$, and α has the minimal polynomial

$$f(x) = x^5 + x^2 + 1 \in \mathbb{F}_2[x].$$

It is known that received word $\bar{w}$ has the syndrome polynomial

$$s(x) = x^2 + x + \beta \text{ , where } \beta = \alpha^2 + \alpha + 1$$

and that $\bar{w}$ has no more than 2 errors. Find the error vector using the EA based decoding procedure.

Problem 7

Let C be binary code with codewords being matrices 7 x 24 with rows from the Golay G_{24} code and columns from the Hamming [7,4] code. It is known that C was used and the received matrix w has 8 errors. It happens that the syndromes for all rows except the 5.th row are non-zero. Prove that the errors can be corrected.

Problem 8

Explain how to construct a binary BCH-code with $n = 31, d = 4$. What is the maximal value of the dimension k that can be obtained by this construction? Why?