Problem 4 (Exam June 2007)

$\mathbb{F}_8$ constructed with $p(x) = 1 + x + x^3$, β a generator for $\mathbb{F}_8^*$

$$g(x) = (1+x)(\beta+x)(\beta^2+x)(\beta^3+x) = x^4 + \beta^2 x^3 + \beta^5 x^2 + \beta^5 x + \beta^6.$$

Gen. matrix
$$G = \begin{bmatrix} \beta^6 & \beta^5 & \beta^5 & \beta^2 & 1 & 0 & 0 \\ 0 & \beta^6 & \beta^5 & \beta^5 & \beta^2 & 1 & 0 \\ 0 & 0 & \beta^6 & \beta^5 & \beta^5 & \beta^2 & 1 \end{bmatrix}$$

Receives: $\beta^5 x + \beta x^2 + \beta^3 x^3 + \beta^4 x^4 + \beta^2 x^5 = y(x)$

Syndroms:

$$S_0 = y(1) = \beta^2$$
$$S_1 = y(\beta) = 0$$
$$S_2 = y(\beta^2) = \beta$$
$$S_3 = y(\beta^3) = \beta^3$$

We get: $\left[\begin{array}{cc|c} \beta^2 & 0 & \beta \\ 0 & \beta & \beta^3 \end{array}\right]$, with solutions $\sigma_0 = \beta^6$, $\sigma_1 = \beta^2$

$$\sigma(x) = x^2 + \sigma_1 x + \sigma_0 = x^2 + \beta^2 x + \beta^6$$

Roots: 1, β^6

position 0 position 6

Find error-magnitude:

$\left[\begin{array}{cc|c} 1 & 1 & \beta^2 \\ 1 & \beta^6 & 0 \end{array}\right]$, with solutions $X_0 = \beta^6$, $X_1 = 1$

So $e(x) = \beta^6 + x^6$

$$\Rightarrow y(x) + e(x) = \beta^6 + \beta^5 x + \beta x^2 + \beta^3 x^3 + \beta^4 x^4 + \beta^2 x^5 + x^6$$