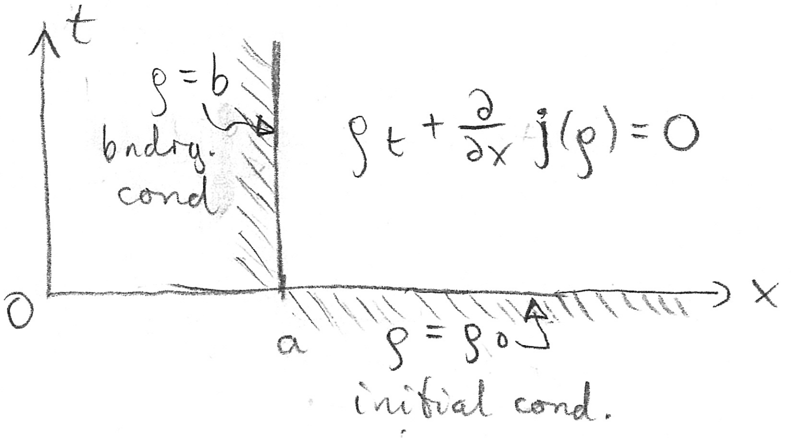
Matmod 15.10.10

A. Boundary value problem:

$$(1)\quad \begin{cases} \rho_t + \frac{\partial}{\partial x} j(\rho) = 0 & x \in [a,\infty),\ t>0 \\ \rho(a,t) = b & x = a,\ t>0 \\ \rho(x,0) = \rho_0(x) & x \in [a,\infty),\ t=0 \end{cases}$$



Fact: Char.'s can start on bndry!

Char. eq'ns: $z(t) = \rho(x(t),t)$

$$\dot{x} = j'(z), \quad \dot{z} = 0$$

Init. cond'ns:

a) Along $t=0, x\geq a$:

$$x(0) = x_0, \quad z(0) = \rho(x(0),0) = \rho_0(x_0) \quad (x_0 \geq a)$$

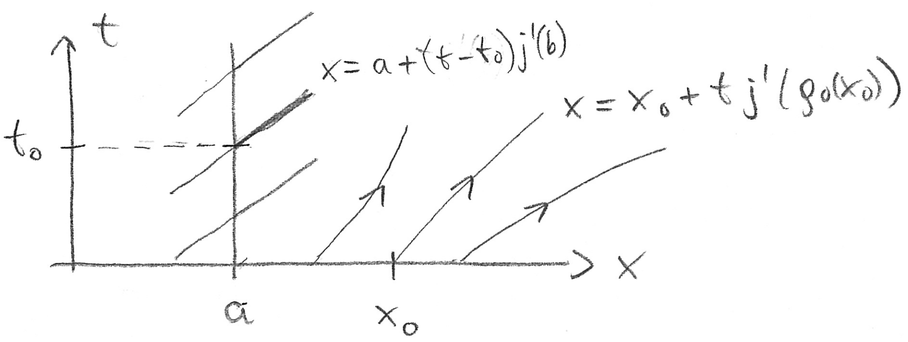
b) Along $x=a, t>0$:

$$x(t_0) = a, \quad z(t_0) = \rho(x(t_0),t_0) = \rho(a,t_0) = b \quad (t_0>0)$$

Sol'ns:

$$\begin{cases} x = x_0 + t j'(\rho_0(x_0)) \\ z = \rho_0(x_0) \end{cases} \qquad x_0 \geq a,\ t \geq 0$$

$$\begin{cases} x = a + (t-t_0) \cdot j'(b) \\ z = b \end{cases} \qquad x_0 = a,\ t > t_0$$



2 possibilities:

1.) Inflow: $j'(b) > 0$

Char's at $x=a$ go into domain

$\Rightarrow$ Sol'n found as before

meth. of char + shocks + ~~fans~~ rarefaction

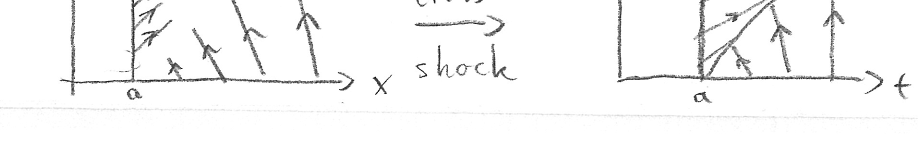
2.) Outflow: $j'(b) \leq 0$

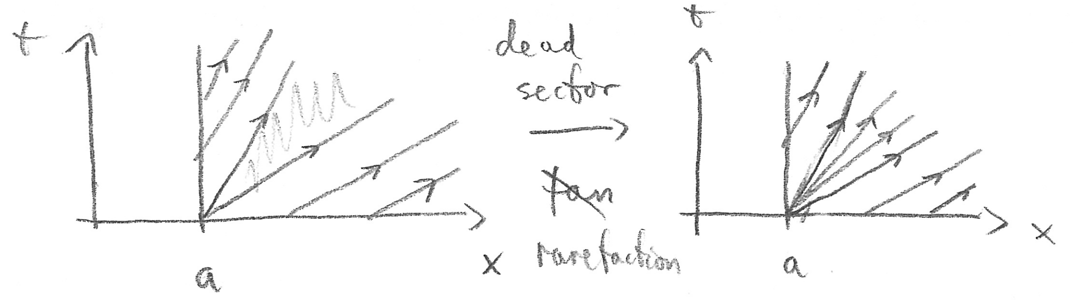
Char's at $x=a$ leave domain

$\Rightarrow$ sol'n ignores bndry-cond. at $x=a$.

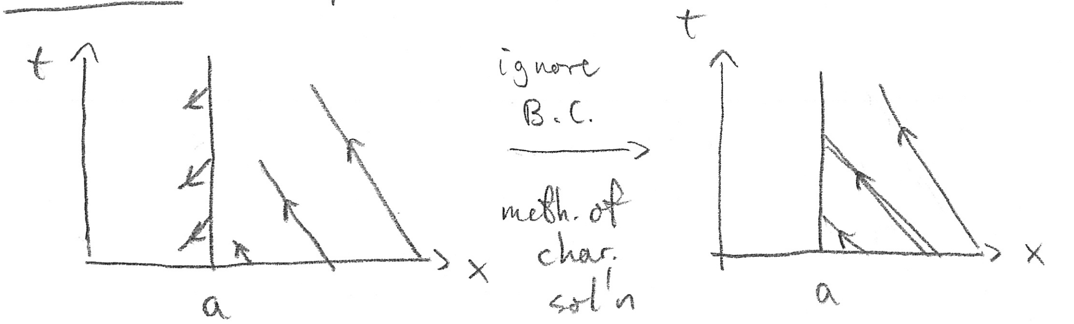
meth. of char. + shocks + ~~fans~~ rarefaction + ignore $x=a$.

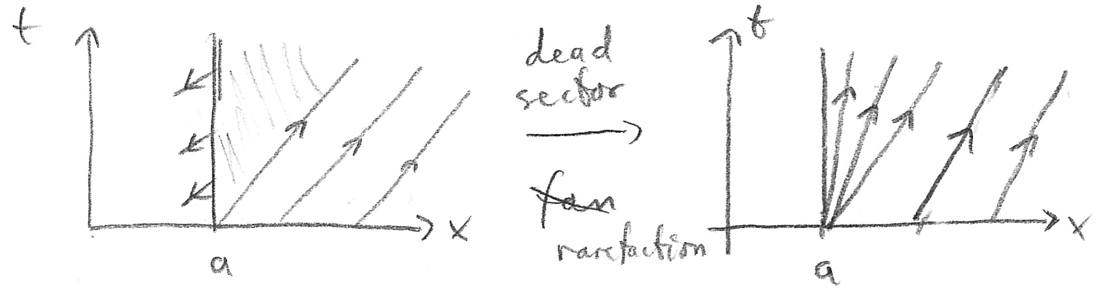
Ex. 1: Inflow





Ex. 2: Outflow





Obs: In Ex. 2, typically $\rho(a^+, t) \neq b$!

B. Flux-condition

$$
(2) \begin{cases} \rho_t + \dfrac{\partial}{\partial x} j(\rho) = 0 \ , & x \geq a, \ t > 0 \\ j(\rho) = b \ , & x = a, \ t > 0 \\ \rho(x,0) = \rho_0(x) \ , & x \geq a, \ t = 0 \end{cases}
$$

Obs: $j = 0$ at $x = a \Rightarrow$ no flux (transport) across $x = a$

Idea: Convert (2) to (1) by solving
$j(\rho) = b$ for ρ
[If many sol'n, select physical one ...]