LAX-FRIEDRICH SCHEME

The scheme presented in [1] relies on a *Riemann solver*. Such solver is meant to give an approximation of the numerical flux at $x = 0$ for the solution of the Rieman problem defined by

$$U_t + F(U)_x = 0 \tag{0.1}$$

with initial condition

$$U(0, x) = \begin{cases} U_- & \text{if } x < 0 \\ U_+ & \text{if } x > 0. \end{cases} \tag{0.2}$$

Here U is a vector in $\mathbb{R}^n$ and $F : \mathbb{R}^n \to \mathbb{R}^n$ a function. It can be proven that, for such solution, the value of the flux at $x = 0$, that is $F(U(t, 0))$, is constant in time. It is usually difficult to compute an exact solution of the Riemann problem. Therefore, it is common to use an approximate solution and we denote by $\mathcal{F}(U_-, U_+)$ a numerical approximation of $F(U(t, 0))$. Still approximate solutions have to be chosen with care in order to get the right physical solution. A simple choice is the Lax-Friedrichs flux, which is given by

$$\mathcal{F}(U_-, U_+) = \frac{1}{2\lambda}(U_- - U_+) + \frac{1}{2}(F(U_-) + F(U_+)) \tag{0.3}$$

where

$$\lambda = \frac{\Delta t}{\Delta x}.$$

The Lax-Friedrich is very *dissipative* and sharp fronts will be rapidly smeared out. Small discretization step will compensate for that. You should then be aware of the Courant-Lax-Friedrichs condition. To learn more on that see [2] or, even better, take the course Nonlinear Partial Differential Equations at NTNU!

References

[1] Emmanuel Audusse et al. "A fast and stable well-balanced scheme with hydrostatic reconstruction for shallow water flows". In: *SIAM Journal on Scientific Computing* 25.6 (2004), pages 2050–2065.

[2] Helge Holden and Nils H Risebro. *Front tracking for hyperbolic conservation laws*. Volume 152. Springer, 2013.

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