Improved kinetic theory of car traffic

1. Modified (scaled) car velocity

   (1) $$v(\rho) = 1 - \rho - \frac{\kappa}{\rho}\frac{\partial \rho}{\partial x}.$$

   Cars slow down to avoid shocks (where $\frac{\partial \rho}{\partial x} \to +\infty$).

2. New conservation law (check!):

   (2) $$\frac{\partial \rho}{\partial t} - \frac{\partial}{\partial x}(\rho(1-\rho)) = \kappa \frac{\partial^2 \rho}{\partial x^2}.$$

   Parabolic equation, smooth solution, shocks become smeared.

   Less smearing for κ small since then $\frac{\partial \rho}{\partial t} - \frac{\partial}{\partial x}(\rho(1-\rho)) \approx 0$.

3. Traveling wave solution (smeared shock): $\rho(x,t) = u(x-at)$

   Insert into (2) + $\lim_{x\to\pm\infty} \rho(x,t) = \rho^{\pm}$ + $\lim_{x\to\pm\infty} \rho_x(x,t) = 0$

   $\underset{\rho_- < \rho_+}{\Rightarrow}$ ODE problem for u, a with solution $a = 1 - \rho^+ - \rho^-$ and

   $$\rho(x,t) = \rho^- + (\rho^+ - \rho^-)\frac{1}{1 + e^{\frac{\rho^- - \rho^+}{\kappa}(x-at)}}$$