Summary of Dimensional Analysis

"Physical relations are (equivalent to) relations between dimensionless combinations"

Recipe:

1. Choose relevant physical quantities $R_1, \ldots, R_m$.

   Conjecture there is *some* relation $\Phi(R_1, \ldots, R_m) = 0$.

2. Find dimension matrix A ($n \times m$) and $r = \mathrm{rank}\, A$.

3. Select r core variables.

   Find $m - r$ dimensionless combinations $\pi_1, \ldots, \pi_{m-r}$.

4. Pi-theorem: $\Phi(R_1, \ldots, R_m) = 0 \;\Leftrightarrow\; \Psi(\pi_1, \ldots, \pi_{m-r}) = 0$

5. Specify Ψ if possible...

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Result:

A dimensionally consistent model $\Psi(\pi_1, \ldots, \pi_{m-r}) = 0$

OBS: Any Ψ gives a dimensionally consistent model!

Advantages:

- Easy to obtain simple models
- Minimize number of variables

Summary of Dimensional Analysis

Questions:

- How to select R_j?
- Many possible π_j's !?
- Ψ is unknown !?

Partial answers:

- Choices based on: physical insight and/or simplicity
- Any relevant fundamental unit must occur in at least 2 R_j's
- $\pi_j = \dfrac{R_{r+j}}{R_1^{\bullet} \cdots R_r^{\bullet}}$ / well-known combinations (Re, ...)
- Ψ ... need extra information/observations ...