# Scaling and non-dimensionalizing

Produce dimensionless variables and equations, with variables and coefficients $\lesssim 1$.

**Scaling a variable $u^*$:** $\boxed{u^* = Uu}$

scaled variable: $u \sim 1,$ $[u] = 1$

scaling constant: $U \sim \max|u^*|,$ $[U] = [u^*]$

**Scaling/nondimensionalizing an equation:**

1. scale all variables in the equation
2. divide the resulting equation by $\sim$ biggest coefficient.

# Finding scales

Scales are combinations of the parameters of the problem

**Recipe:**

1. Introduce scaled variables into all equations and constraints (scales to be determined): $x^* = Xx, t^* = Tt, \frac{dx^*}{dt^*} = \frac{X}{T}\frac{dx}{dt}$...
2. Scaling assumption: Scaled variables and derivatives $\sim 1$:
   $t, x(t), x'(t), \cdots \sim 1$.
3. Determine scales $X, T, \ldots$ by
   a) determining directly $\max|t^*|$, $\max|x^*|$, ..., or
   b) balance 2 dominating/biggest terms in equation, or
   c) solve a reduced problem to find estimates

# Remarks on scaling

**When 2 dominating terms balances:**

1. Their coefficients are equal and $\sim$ biggest in equation.
2. Dividing the (scaled) equation by this coefficient
   $\Rightarrow$ all variables and coefficients becomes
   a. dimensionless,
   b. $\lesssim 1$, and
   c. 2 coefficients $= 1$.

**Different situations $\Rightarrow$ different scales:**

$\max|x^*|$, $\max|t^*|$, $\max|u^*|$, etc., and hence the dominating terms in the equation depend on the situation.

# Remarks on scaling

**Typical time scale** for $u^*(t^*)$:

$$T = \frac{\max|u^*|}{\max\left|\frac{du^*}{dt^*}\right|}$$

**Advantages of scaling:**

- minimize the number of parameters/coefficients,
- normalize all variables and coefficients,
- reduce round-off errors in numerical calculations,
- makes small terms visible
  $\Rightarrow$ easy to do approximations/perturbation.