

Regular Perturbation

Given scaled(!) equation: $\ddot{x} = -\frac{1}{(1 + \varepsilon x)^2}$, $0 < \varepsilon \ll 1$.

1. **Perturbation Assumption:** $x = x_0 + \varepsilon x_1 + \varepsilon^2 x_2 + \dots$
2. Insert into equation, expand as **power series in ε** :

$$\begin{aligned}\ddot{x}_0 + \varepsilon \ddot{x}_1 + \varepsilon^2 \ddot{x}_2 + \dots &= -\frac{1}{\left(1 + \varepsilon(x_0 + \varepsilon x_1 + \dots)\right)^2} \\ &= -1 + 2\varepsilon(x_0 + \varepsilon x_1 + \dots) - 3\varepsilon^2(x_0 + \varepsilon x_1 + \dots)^2 + \dots \\ &= -1 + \varepsilon 2x_0 + \varepsilon^2(2x_1 - 3x_0^2) + \dots\end{aligned}$$

3. Equate terms of **same order in ε** to find equations for x_0, x_1, \dots :

$$O(1): \quad \ddot{x}_0 = -1$$

$$O(\varepsilon): \quad \ddot{x}_1 = 2x_0$$

$$O(\varepsilon^2): \quad \ddot{x}_2 = 2x_1 - 3x_0^2$$

5. Solve these equations **recursively** for x_0, x_1, x_2, \dots .