Conservation laws in Continuum Mechanics

Markus Grasmair

Department of Mathematics,
Norwegian University of Science and Technology,
Trondheim, Norway

Trondheim,
October 17, 2018

Goal of the lecture

- Conservation laws where the conserved quantity is *particle based*, not region based.
- Typical examples:
  - Conservation of momentum.
  - Conservation of energy.
- Equations for the conservation of momentum:
  - General situation independent of state of matter.
  - Particular models for fluids.
- (Derivation of Navier–Stokes equations.)

Euler and Lagrange formulations

Euler formulation:

- Fix a control volume and formulate the conservation law within this region.
- Approach we have used in traffic modelling.
- Obtain good description of the kinematics.
- "Standard" numerical methods can be applied.

Lagrange formulation:

- Select a collection of particles and follow these particlese as they move; formulate the conservation law for the *moving* particles.
- Conservation of momentum and energy hold only for individual particles.
- Numerical solution can become difficult.

Switch between formulations: *Reynold's transport theorem*.

Conservation of momentum in continuum mechanics

Newton's second law:

- Change of momentum = sum of all forces.

Forces acting on a collection of particles:

- Body forces: long range forces acting on the particles (gravity, electro-magnetic forces,...).
- Surface forces: forces between adjacent particles (friction).

Surface forces are encoded by means of the *stress tensor*. Microscopic properties of the material determine large scale structural properties of the tress tensor.

Literature: Kompendium, pp. 201–207.
More in depth discussion: Lin & Segel, chap. 13–15.