Scaling (and non-dimensionalisation)

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Scaling

Goal: reformulate relations in such a way that all relevant quantities are of size $\lesssim 1$.

- Unscaled variable u^*.
- Write

$$u^* = Uu$$

  with:
  u ... scaled variable,
  U ... characteristic scale (size) of u^*.
- Good choice of scale is such that $u \sim 1$ *within the region of interest*.

Advantages:

- Reduce the number of parameters/coefficients.
- Normalise all variables (stabilise numerics).
- Identify small (and very small) terms.

Finding scales

Typical scaling for a time dependent function $u^*(t^*)$:

- Scale u^* to values $\lesssim 1$:

$$u^* = uU \text{ with } U \sim \max_{t^*} |u^*(t^*)|.$$

- Scale time t^* such that velocities (derivatives) are $\lesssim 1$:

$$t^* = tT \text{ with } T \sim \frac{\max_{t^*} |u^*(t^*)|}{\max_{t^*} \left|\frac{du^*}{dt^*}(t^*)\right|}.$$

- Maxima are taken over the region of interest.
- Different situations lead to different scalings.