

# Singular perturbations — enzyme kinetics

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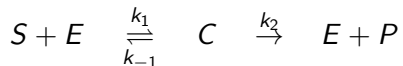
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# Chemical reaction

- $S$  ... substrate with concentration  $s$ ,
- $E$  ... enzyme with concentration  $e$ ,
- $C$  ... complex with concentration  $c$ ,
- $P$  ... product with concentration  $p$ .

Processes at work:



# System of equations

Chemical reaction can be modelled by the system

$$\begin{aligned}\frac{ds}{dt} &= -k_1se + k_{-1}c, \\ \frac{de}{dt} &= -k_1se + (k_{-1} + k_2)c, \\ \frac{dc}{dt} &= k_1se - (k_{-1} + k_2)c, \\ \frac{dp}{dt} &= k_2c,\end{aligned}$$

with initial conditions

$$\begin{aligned}e(0) &= \bar{e} & s(0) &= \bar{s}, \\ c(0) &= 0 & p(0) &= 0.\end{aligned}$$

# Straightforward simplification

- Conservation of enzymes:

$$e + c = e(0) + c(0) = \bar{e}.$$

- Conservation of substrate:

$$s + c + p = s(0) + c(0) + p(0) = \bar{s}.$$

Can explicitly compute

$$e(t) = \bar{e} - c(t),$$

$$p(t) = \bar{s} - \bar{e} - s(t) + e(t).$$

## Simplified system

After elimination of  $e$  and  $p$ , we obtain

$$\begin{aligned}\frac{ds}{dt} &= -k_1 \bar{e}s + k_1 sc + k_{-1}c, \\ \frac{dc}{dt} &= k_1 \bar{e}s - (k_{-1} + k_2)c - k_1 sc,\end{aligned}$$

with initial conditions

$$s(0) = \bar{s}, \quad c(0) = 0.$$

Assumption:

- $\bar{e} \ll \bar{s}$  — quantity of enzymes driving the process is much smaller than the quantity of substrate.