TMA 4195 Mathematical Modelling

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August 19, 2019

My background

- Lecturer: Dag Wessel-Berg
- PhD from NTH, 1994, algebraic geometry & integrable non-linear partial differential equations
- Permanent lecturer at NTNU since 1988
- Research Scientist/Senior Scientist at SINTEF 1992-2017
- Associate Professor NTNU 2017-
- Research: CO_2 storage, stability analysis

… and this is the first time I teach this course!

Course information

- wiki.math.ntnu.no/tma4195/2019h/start

Project

- Grading: Final exam (75%) and project (25%).
- Literature: See wiki page
- Exercises: Are not mandatory — **but strongly recommended.**
- Project: Week 44-46, group work. Topic: CO2 storage. Responsible: Xavier Raynaud

NTNU

Mathematical modelling

. . . can be roughly described as the art of translating practical problems from applied areas into mathematical formulations, and using these formulations together with theoretical and numerical methods in order to get further insight into the problem and answer concrete practical questions.

Typical procedure:

1. Identification of the problem and the relevant quantities.
2. Formulation of the relation between these quantities.
3. Simplification and model reduction.
4. Theoretical analysis and/or numerical approximation.
5. Comparison of the results with real data.
6. Revision and refinement of the model

Main topics

1 Dimensional analysis

- Simple mathematical models.
- Reduction of the number of variables.
- Simple methods for basic model validation.

2 Scaling
- Systematization of problem analysis.
- Identification of important and negligible quantities.
- Possible model simplification and dimension reduction.

3 Perturbation analysis
- Approximation of equations and solutions of equations with very
- small/large quantities.

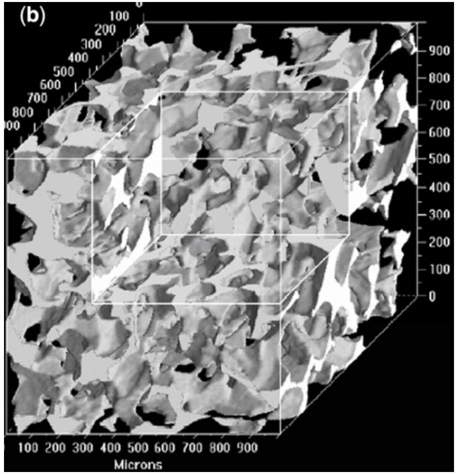
4 Specific methods and models
- Simple dynamical models:
- Very common models; easy to understand, analyze, and implement.
- Conservation laws:
- Basic principle for the derivation of various models.

Dimensional Analysis and Scaling

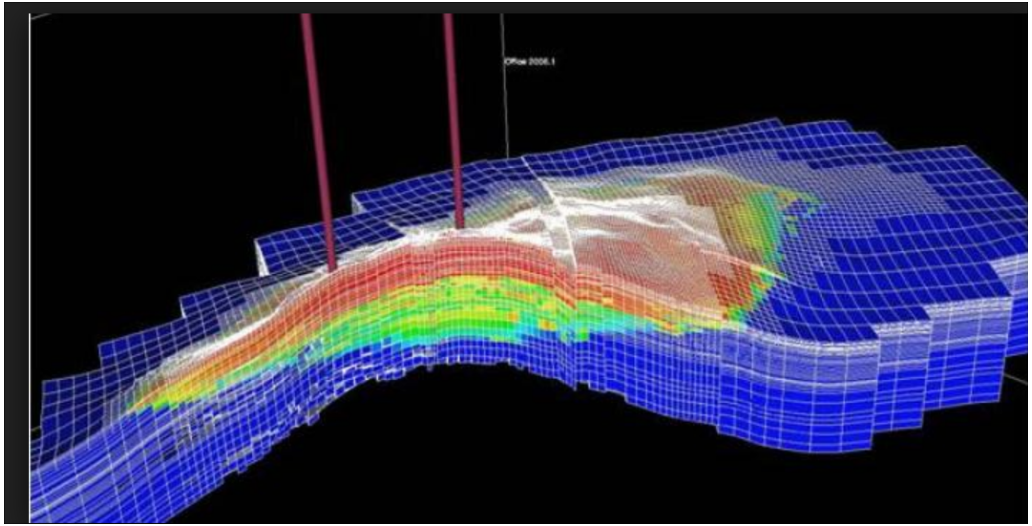
Can small models tell us anything about large models?

Why are aunts so «strong»?

Scale of description



«Upscaling»



Multiphase flow in porous media at micrometer scale

Predictive(?) reservoir model at kilometer scale!

SI (système international)

SI base units[4]:23[5][6]

Unit name	Unit symbol	Dimension symbol	Quantity name	Definition
second [n 1]	s	T	time	The duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium-133 atom.
metre	m	L	length	The distance travelled by light in vacuum in $\frac{1}{299\,792\,458}$ second.
kilogram [n 2]	kg	M	mass	The kilogram is defined by setting the Planck constant *h* exactly to $6.626\,070\,15 \times 10^{-34}$ J⋅s (J = kg⋅m^2⋅s^{-2}), given the definitions of the metre and the second.[1]
ampere	A	I	electric current	The flow of $\frac{1}{1.602\,176\,634 \times 10^{-19}}$ times the elementary charge *e* per second.
kelvin	K	Θ	thermodynamic temperature	The kelvin is defined by setting the fixed numerical value of the Boltzmann constant *k* to $1.380\,649 \times 10^{-23}$ J⋅K^{-1}, (J = kg⋅m^2⋅s^{-2}), given the definition of the kilogram, the metre, and the second.
mole	mol	N	amount of substance	The amount of substance of exactly $6.022\,140\,76 \times 10^{23}$ elementary entities.[n 3] This number is the fixed numerical value of the Avogadro constant, N_A, when expressed in the unit mol^{-1} and is called the Avogadro number.
candela	cd	J	luminous intensity	The luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 5.4×10^{14} hertz and that has a radiant intensity in that direction of $\frac{1}{683}$ watt per steradian.

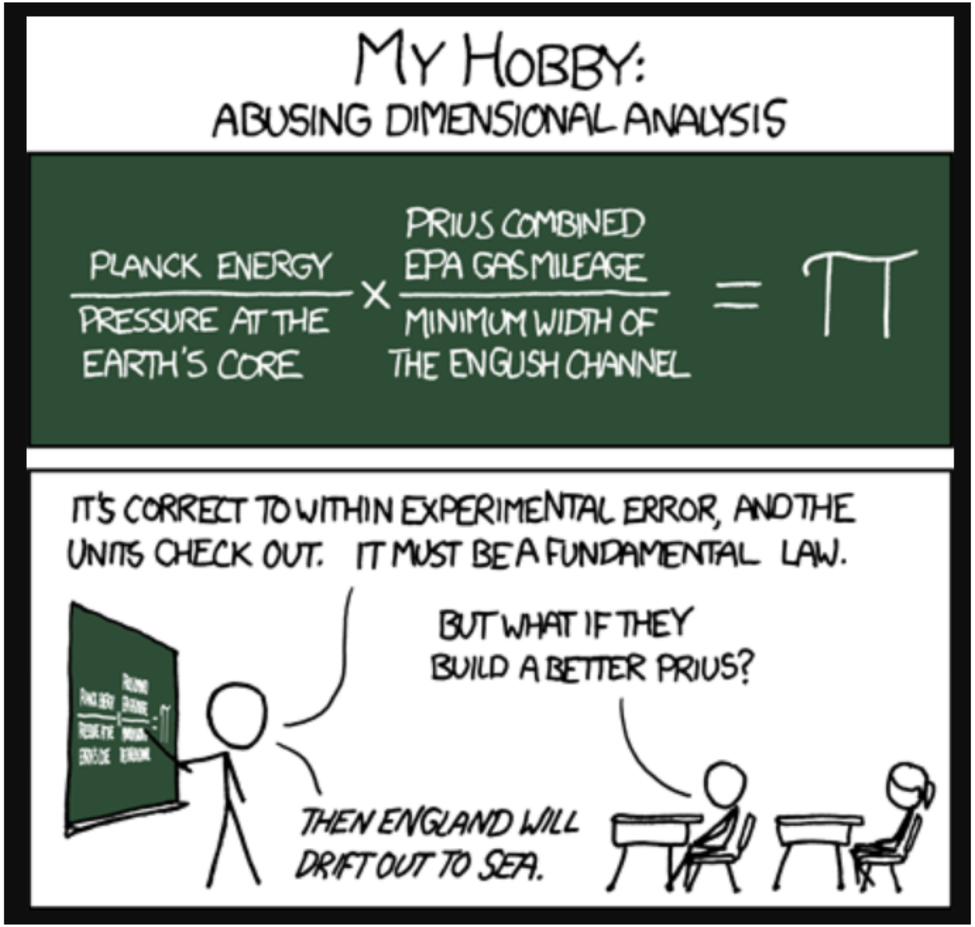
Notes

1. ^ Within the context of the SI, the second is the coherent base unit of time, and is used in the definitions of derived units. The name "second" historically arose as being the 2nd-level sexagesimal division ($1/60^2$) of some quantity, the hour in this case, which the SI classifies as an "accepted" unit along with its first-level sexagesimal division the minute.
2. ^ Despite the prefix "kilo-", the kilogram is the coherent base unit of mass, and is used in the definitions of derived units. Nonetheless, prefixes for the unit of mass are determined as if the gram were the base unit.
3. ^ When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles.

Dimensional analysis, recipe

- Assume there is a relation $f\left(R_1, R_2, .., R_n\right) = 0$
- Form the $m \times n$ – «dimension matrix» A, $\operatorname{rank}\left(A\right) = k$
- Pick k «core» variables $R_{j_1}, R_{j_2}, .., R_{j_k}$ corresponding to column vectors in A giving a basis for $\operatorname{col}\left(A\right)$
- Using $R_{j_1}, R_{j_2}, .., R_{j_k}$, form $n-k$ dimensionless variables

  $\pi_1, \pi_2, .., \pi_{n-k}$
- Then there exist a relation $g\left(\pi_1, \pi_2, .., \pi_{n-k}\right) = 0$


MY HOBBY:
ABUSING DIMENSIONAL ANALYSIS
PLANCK ENERGY / PRESSURE AT THE EARTH'S CORE × PRIUS COMBINED EPA GAS MILEAGE / MINIMUM WIDTH OF THE ENGLISH CHANNEL = π
IT'S CORRECT TO WITHIN EXPERIMENTAL ERROR, AND THE UNITS CHECK OUT. IT MUST BE A FUNDAMENTAL LAW.
BUT WHAT IF THEY BUILD A BETTER PRIUS?
THEN ENGLAND WILL DRIFT OUT TO SEA.