3 physical regimes for sinking ball

Subtitle

Misprint: v_{FF}^2

Case	Characteristics	Length scale	Time scale	Equation	Parameters
A	$v_0 \ll v_{FF}$	L	L/v_0	$\varepsilon\ddot{x} + \dot{x} = 1$	$\varepsilon = 2\frac{v_0^2}{v_{FF}}$, $\mu = \frac{V}{v_0}$
B	$v_0 \gg v_{FF}, V < v_{FF}$	L	L/v_{FF}	$2\ddot{x} + \varepsilon\dot{x} = 1$	$\varepsilon = \frac{v_{FF}}{v_0}$, $\mu = v/v_{FF}$
C	$V \gg v_0$	mv/k	m/k	$\ddot{x} + \dot{x} = \varepsilon$	$\varepsilon = \frac{v_0}{v}$, $\mu = 1$

A: High friction and not «too high» V

B: Low friction

C: High friction, high V

C: Parameter

$$L_D = \frac{L}{\left(\frac{mv}{k}\right)} = \frac{kL}{mv}$$

Dimensionless depth

Diffusion

Diffusion acts to even out differences

$$\frac{\partial u^*}{\partial t^*} = D\frac{\partial^2 u^*}{\partial x^{*2}},\ 0 \le x \le L,\ t \ge 0$$

$$x^* = Lx,\ t^* = Tt,\ u^* = Uu \quad \Rightarrow \frac{\partial u}{\partial t} = \left(\frac{TD}{L^2}\right)\frac{\partial^2 u}{\partial x^2}$$

$$T = \frac{L^2}{D}$$ time scale for this process

Diffusion of momentum

The (dynamic) viscosity of a fluid is a measure for the (transversal) diffusion of momentum, and is essentially expressed by

$$\rho \frac{\partial u^*}{\partial t^*} = \mu \frac{\partial^2 u^*}{\partial x^{*2}} \Leftrightarrow \frac{\partial u^*}{\partial t^*} = \nu \frac{\partial^2 u^*}{\partial x^{*2}}$$

Thus, the kinematic viscosity,

$$\nu = \frac{\mu}{\rho}$$

is the diffusion coefficient for velocity

Turbulence

Large whirlpools generate smaller whirlpools, again triggering smaller whirlpools, etc., until whirlpools become small enough for friction (shear forces) to convert kinetic energy to thermal energy.

The size and velocity of these smallest whirlpools are called *Kolmogorov's microscales* in turbulence theory.

A woodcut print by Katsushika Hokusai,