TMA4205 - Autumn 2012

Assignment 4

Problem 1. (From Saad, chap 5) Consider the matrix

$$A = \begin{bmatrix} 1 & -6 & 0 \\ 6 & 2 & 3 \\ 0 & 3 & 2 \end{bmatrix}$$

(a) Find a rectangle in the complex plane which contains all the eigenvalues of A without actually computing the eigenvalues.

(b) Can one assert that the MR iteration always converges for a linear system with matrix A?

Problem 2. Yet again we return to solving the one dimensional Poisson problem

$$\begin{aligned} -\frac{\mathrm{d}^2 u}{\mathrm{d}x^2} &= 4\pi^2 \sin 2\pi x, \quad && x \in [0,1], \\ u &= 0, && x \in \{0,1\} \end{aligned}$$

that was discussed in Assignment 1. Let us again use the finite difference method on a uniform mesh with step-size $h = 1/n$, and grid points $x_j = jh$, $j = 0, \dots, n$.. The discretized equations can be expressed as $A\,u = b$ where A represents the discrete Laplacian. We already studied how to obtain the exact eigenvalues of the matrix A. We now want to solve this linear system of equations by three different iterative methods: Jacobi iteration, Steepest descent, and minimal residual (MR) iteration.

(a) Suppose that we want to reduce the initial error by 5 orders of magnitude. Estimate the number of iterations required in the Jacobi method and with the Steepest descent method.

(b) Suppose that we want to reduce the initial residual in the solution by 5 orders of magnitude. Estimate the number of iterations required in the minimal residual iteration.

(c) Discuss the computational cost (complexity) in the three iterative methods.

(d) Which are the relative advantages in the various methods (if they exist), both in terms of solving the Poisson problem, and in a more general context solving linear systems?