TMA4205 - Autumn 2012
Assignment 7

Problem 1

In this exercise we consider the restriction operator I_h^{2h} (based on full weighting) used in the multigrid algorithm. We discussed this operator in class in the context of solving the one-dimensional Poisson problem.

(a) What is the rank of this operator?

(b) Show that the null space of the restriction operator I_h^{2h} has a basis consisting of vectors of the form $(0, \dots, 0, -1, 2, -1, 0, \dots, 0)^T$. Count the number of such vectors and show that $\dim(\text{null}(I_h^{2h})) = \frac{n}{2}$.

(c) Do the basis vectors for the null space of I_h^{2h} correspond to oscillatory modes?

(d) Use MATLAB to compute the SVD of I_h^{2h} in the case $n = 8$, i.e., $I_h^{2h} \in \mathbb{R}^{3\times 7}$; use the built-in routine for this. Is the result from (a) consistent with the result from the SVD? What does the SVD say about $\dim(\text{null}(I_h^{2h}))$?

Problem 2

Exercise 24.1 in Trefethen and Bau.

Problem 3

Consider the matrix

$$A = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}.$$

(a) Using Householder reflectors, compute (by hand) the QR-factorization of A.

(b) Calculate the eigenvalues and eigenvectors of the matrix $A^T A$.

(c) Use your results in (b) to compute (by hand calculations) the SVD of A.

(d) Find the 1-, 2-, ∞- and Frobenius-norms of A?