Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4205 Numerical Linear Algebra
Fall 2013

Exercise set 6

$\boxed{1}$ Consider the matrix

$$A = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}.$$

a) Using Householder reflectors, compute (by hand) the QR factorization of A.

b) Calculate the eigenvalues and eigenvectors of the matrix $A^{\mathrm{T}}A$.

c) Use your results in **b)** to compute (by hand) the SVD of A.

d) Find the 1-, 2-, ∞- and Frobenius norms of A.

$\boxed{2}$ For each of the following, show that the statement is correct, or give a counter-example. If nothing else is written, assume that $A \in \mathbb{C}^{m\times m}$.

a) If λ is an eigenvalue of A and $\mu \in \mathbb{C}$, then $\lambda - \mu$ is an eigenvalue of $A - \mu\mathrm{I}$.

b) If A is real and λ is an eigenvalue of A, then $-\lambda$ is an eigenvalue of A.

c) If A is real and λ is an eigenvalue of A, then $\bar{\lambda}$ is an eigenvalue of A.

d) If λ is an eigenvalue of A and A is nonsingular, then λ^{-1} is an eigenvalue of A^{-1}.

e) If all the eigenvalues of A are zero, then $A = 0$.

f) If A is Hermitian and λ is an eigenvalue of A, then $|\lambda|$ is a singular value of A.

g) If A is diagonalizable and all eigenvalues are equal, then A is diagonal.