



- 1 Consider the matrix

$$A = \begin{bmatrix} 1 & -6 & 0 \\ 6 & 2 & 3 \\ 0 & 3 & 2 \end{bmatrix}$$

- a) Find a rectangle or a square in the complex plane which contains all the eigenvalues of  $A$  without actually computing the eigenvalues.
- b) Can one assert that the MR iteration always converges for a linear system with matrix  $A$ ?

- 2 Yet again we return to solving the one dimensional Poisson problem that was discussed in the first exercise set.

$$\begin{aligned} -\frac{d^2u}{dx^2} &= 4\pi^2 \sin(2\pi x), & x \in [0, 1], \\ u &= 0, & x \in \{0, 1\}. \end{aligned}$$

Let us again use the finite difference method on a uniform grid with step-size  $h = 1/n$  and grid points  $x_j = jh$ . The discretized equations can be expressed as  $Au = b$  where  $A$  represents the discrete Laplacian. We now want to solve this system of linear equations by three different iterative methods: Jacobi iteration, steepest descent (SD), and minimal residual (MR) iteration.

- a) Suppose that we want to reduce the initial error by 5 orders of magnitude. Estimate the number of iterations required in the Jacobi method and with SD.
- b) Suppose that we want to reduce the initial residual by 5 orders of magnitude. Estimate the number of iterations required in MR.
- c) Discuss the computational cost (complexity) of the three iterative methods.
- d) What are the relative advantages (if they exist) of the various methods, both in terms of solving the Poisson problem, and in the more general context of solving linear systems?