Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4205 Numerical Linear Algebra Fall 2014

Solutions to exercise set 4

$\boxed{1}$ **Saad, Exercise 5.3** We first consider the system $Ax = b$ with $\mathscr{L} = A\mathscr{K}$. Let

$$\begin{aligned} \mathscr{K} &= \operatorname{span}\{v_1, \dots, v_m\}, \\ V_m &= [v_1 \mid \cdots \mid v_m] \in \mathbb{R}^{n \times m}. \end{aligned}$$

In the first case, the approximate solution $\tilde{x}$ to $Ax = b$ must satisfy

$$\begin{aligned} \tilde{x} - x_0 &\in \mathscr{K}, \\ (AV_m)^{\mathrm{T}}(b - A\tilde{x}) &= 0. \end{aligned}$$

Next, we consider the system $A^{\mathrm{T}}Ax = A^{\mathrm{T}}b$ with $\mathscr{L} = \mathscr{K}$ (*orthogonal* projection). The approximate solution must now satisfy

$$\begin{aligned} \tilde{x} - x_0 &\in \mathscr{K}, \\ V_m^{\mathrm{T}}(A^{\mathrm{T}}b - A^{\mathrm{T}}A\tilde{x}) &= 0 \\ &\Downarrow \\ (AV_m)^{\mathrm{T}}(b - A\tilde{x}) &= 0. \end{aligned}$$

Hence, we get the same system $(AV_m)^{\mathrm{T}}A\tilde{x} = (AV_m)^{\mathrm{T}}b$ or $(AV_m)^{\mathrm{T}}A\delta = (AV_m)^{\mathrm{T}}r_0$, where $\tilde{x} = x_0 + \delta$. This shows that the two methods are equivalent.