Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4205 Numerical Linear Algebra Fall 2015

Exercise set 4

1 Yet again we return to solving the one dimensional Poisson problem that was discussed in the first exercise set.

$$
\begin{aligned}
-\frac{\mathrm{d}^2 u}{\mathrm{d}x^2} &= 4\pi^2 \sin(2\pi x), && x \in [0,1],\\
u &= 0, && x \in \{0,1\}.
\end{aligned}
$$

Let us again use the finite difference method on a uniform grid with step-size $h = 1/n$ and grid points $x_j = jh$. The discretized equations can be expressed as $Au = b$ where A represents the discrete Laplacian. We now want to solve this system of linear equations by three different iterative methods: Jacobi iteration, steepest descent (SD), and minimal residual (MR) iteration.

a) Suppose that we want to reduce the initial error by 5 orders of magnitude. Estimate the number of iterations required to achive this using Jacobi and steepest descent (SD) algorithms.

b) Suppose that we want to reduce the initial residual by 5 orders of magnitude. Estimate the number of iterations required in MR.

c) Discuss the computational cost (complexity) of the three iterative methods.

d) What are the relative advantages (if they exist) of the various methods, both in terms of solving the Poisson problem, and in the more general context of solving linear systems?

2 **Saad, Exercise 5.3** In Section 5.3.3, it was shown that using a one-dimensional projection method with $\mathcal{K} = \operatorname{span}\{A^{\mathrm{T}} r\}$ and $\mathcal{L} = \operatorname{span}\{AA^{\mathrm{T}} r\}$ is equivalent to using the steepest descent method on the normal equations $A^{\mathrm{T}}Ax = A^{\mathrm{T}}b$ for a matrix $A \in \mathbb{R}^{n\times n}$, $b \in \mathbb{R}^n$.

Show that a Galerkin method for $A^{\mathrm{T}}Ax = A^{\mathrm{T}}b$ with solution space $\mathcal{K}$ $[= \mathcal{L}]$ is equivalent to applying a Petrov–Galerkin method to the system $Ax = b$ with the same solution space $\mathcal{K}$ and $\mathcal{L} = A\mathcal{K}$.