TMA4205 Numerical Linear Algebra Fall 2015

Norwegian University of Science and Technology
Department of Mathematical Sciences

Exercise set 8

1 Consider a system of equations $Ax = b$ with a non-singular matrix $A \in \mathbb{R}^{n\times n}$. Let $A = A_1 - A_2$ be a matrix splitting of A with $\det A_1 \neq 0$. We will use a matrix-splitting algorithm as a preconditioner M for a Krylov algorithm, that is, $y = M^{-1}v$ is computed by running the matrix-splitting algorithm for $Ay = v$ starting with $y^0 = 0$.

a) Suppose that we stop the matrix-splitting algorithm after ν iterations. Write down the explicit formula for M^{-1} in terms of A_1^{-1}, A_2.

b) Let A_1 be a symmetric positive definite matrix. For which $\nu \geq 1$ can we *guarantee* that M^{-1} will be a symmetric positive definite matrix?

c) Consider a model tri-diagonal matrix A resulting from 3-point finite difference discretization of the Lalpacian in 1D on a regular grid. Prove that left preconditioning based on 1 iteration of Jacobi has no effect on the speed of convergence of the Krylov based methods.

2 Exercise 1 demonstrates that aperforming a *fixed* number ν of steps of a matrix-splitting algorithm generates a constant preconditioner. That is, applying such a strategy to a vector v is mathematically equivalent to a matrix-vector multiplication $M^{-1}v$ for some fixed matrix M^{-1}.

Show that this is not the case for Krylov-based methods. Namely, let y_1 be a result of applying 1 iteration of the conjugate gradient method to $Ay = v$, starting with $y_0 = 0$. (We make a blanket assumption that A is SPD here.) Show that y_1 cannot be represented as a product $M^{-1}v$, where M^{-1} is independent from v, if A has at least two distinct eigenvalues.

3 Consider a 2×2 real matrix

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$$

Find the left (B_L) and right (B_R) sparse approximate inverses A, which has the same sparsity pattern as A. That is, solve numerically the least squares problems

$$\min_{B_L} \frac{1}{2}\|B_L A - I\|_F^2,$$

and

$$\min_{B_R} \frac{1}{2}\|A B_R - I\|_F^2.$$