Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4205 Numerical Linear Algebra Fall 2015

Solutions to exercise set 5

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4 Given the data, the "naïve" basis for Krylov subspace is orthonormal; in fact it is the canonical basis in $\mathbb{R}^5$: $A^k r_0 = A^k b = A^k e_1 = e_{k+1}$, $k = 0, \ldots, 4$. Therefore $V_m = [e_1, \ldots, e_m]$ after m steps of Arnoldi iteration and $H_m = V_m^{\mathrm{T}} A V_m$ is obtained by taking first m rows/columns from A.

Note that for $m = 1, \ldots, 4$ the matrices H_m are singular with the first row being identically equal to zero. As a result, it is impossible to solve the equations $H_m y_m = \beta e_1$, $m = 1, \ldots, 4$, appearing in FOM algorithm (in our case $\beta = \|r_0\|_2 = 1$).

For the last iteration we have $H_5 = A$, $V_5 = I$, $x_5 = y_5 = e_5$.