TMA4205 Numerical Linear Algebra
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Exercise set 3

$\boxed{1}$ Consider the matrix

$$A = \begin{bmatrix} 1 & -6 & 0 \\ 6 & 2 & 3 \\ 0 & 3 & 2 \end{bmatrix}$$

a) Use Gerschgorin's theorem in order to estimate the eigenvalues of the matrix A.

b) Can one assert that the MR iteration always converges for a linear system with matrix A?

$\boxed{2}$ Yet again we return to solving the one dimensional Poisson problem that was discussed in the first exercise set.

$$\begin{aligned} -\frac{\partial^2 u}{\partial x^2} &= 4\pi^2 \sin(2\pi x), \quad && x \in [0,1], \\ u &= 0, && x \in \{0,1\}. \end{aligned}$$

Let us again use the finite difference method on a uniform grid with step-size $h = 1/n$ and grid points $x_j = jh$. The discretized equations can be expressed as $Au = b$ where A represents the discrete Laplacian. We now want to solve this system of linear equations by three different iterative methods: the Jacobi iteration, steepest descent (SD), and minimal residual (MR) iteration.

a) Suppose that we want to reduce the initial error by 5 orders of magnitude. Estimate the number of iterations required in the Jacobi method and with SD.

b) Suppose that we want to reduce the initial residual by 5 orders of magnitude. Estimate the number of iterations required in MR.

c) Discuss the computational cost (complexity) of the three iterative methods.

d) What are the relative advantages (if they exist) of the various methods, both in terms of solving the Poisson problem, and in the more general context of solving linear systems?