Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4205 Numerical Linear Algebra
Fall 2016

Exercise set 4

1 **Saad, Exercise 5.3** In Section 5.3.3, it was shown that using a one-dimensional projection method with $\mathcal{K} = \text{span}\{A^T r\}$ and $\mathcal{L} = \text{span}\{AA^T r\}$ is equivalent to using the steepest descent method on the normal equations $A^T Ax = A^T b$ for a matrix $A \in \mathbb{R}^{n\times n}$, $b \in \mathbb{R}^n$.

Show that an orthogonal projection method with solution space $\mathcal{K} = \mathcal{L}$ for solving the equation $A^T Ax = A^T b$ is equivalent to applying a projection method onto $\mathcal{K}$ orthogonally to $\mathcal{L} = A\mathcal{K}$ for the problem $Ax = b$.

2 Algorithm 6.1 in Saad is implemented in the MATLAB-function `arnoldi_gs.m`. This algorithm constructs an orthogonal basis for the Krylov subspace $\mathcal{K}_m(A, v)$ based on a classical Gram–Schmidt procedure. Test this function on the matrix A generated by `poisson2.m` (use sparse matrices) for different values of m and $N = n^2$. For instance, choose $N = 100$, $v = e_1$, and $m = 10, 20, 30, 40, 50$.

a) Test to what extent the relation $V_m^T AV_m = H_m$ from Proposition 6.5 in Saad is fulfilled. Also check if the vectors $v_1, \ldots, v_m$ really are orthonormal, i.e., check whether $V_m^T V_m = I_m$ (exactly).

b) Modify the function `arnoldi_gs.m` such that it uses modified Gram–Schmidt. Repeat the experiments from the previous question.

3 Assume that a real matrix A is anti-symmetric, that is, $A^T = -A$. Explain the structure of the Hessenberg matrix H_n resulting from Arnoldi process in this case. Explain how this structure can be utilized for an efficient implementation of the Arnoldi process in this case.