TMA4205 Numerical Linear Algebra
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Exercise set 6

$\boxed{1}$ Compute the (reduced) singular value decomposition and the pseudoinverse of the matrix

$$A = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}.$$

$\boxed{2}$ Assume that $A \in \mathbb{R}^{n\times n}$ is skew-symmetric. Show that the singular values of A are precisely the absolute values of the eigenvalues of A.

$\boxed{3}$ Compute the (reduced) singular value decomposition of the matrix

$$A = \begin{pmatrix} 10 & 10 \\ -1 & 7 \\ 5 & 5 \\ -2 & 14 \end{pmatrix}.$$

Additionally, compute the pseudoinverse $A^\dagger$ of A and use it in order to solve the least squares problem

$$\min_{x\in\mathbb{R}^2} \frac{1}{2}\|Ax-b\|_2^2 \qquad \text{where } b = \begin{pmatrix} 7 \\ -5 \\ 1 \\ 1 \end{pmatrix}$$

$\boxed{4}$ Compute the pseudoinverse of the matrix

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}.$$

Using this particular matrix, show that the pseudoinverse of a matrix does not necessarily satisfy the relation $(A^\dagger)^2 = (A^2)^\dagger$.

$\boxed{5}$ Perform two steps of the the Rayleigh quotient iteration with starting vector $v^{(0)} = (1,0,0)^T$ for approximating an eigenvalue and eigenvector of the matrix

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}.$$

$\boxed{6}$ Perform one step of the QR-iteration with shift $\mu = 2$ in order to find the eigenvalues of the matrix

$$A = \begin{pmatrix} 6 & 3 \\ 3 & 2 \end{pmatrix}.$$

$\boxed{7}$ Assume that you apply one step of the QR-iteration with shift μ in order to find the eigenvalues of a matrix A, and that this shift is actually equal to one of the eigenvalues of A. How can you easily detect this situation based on the QR-decomposition of the shifted matrix?