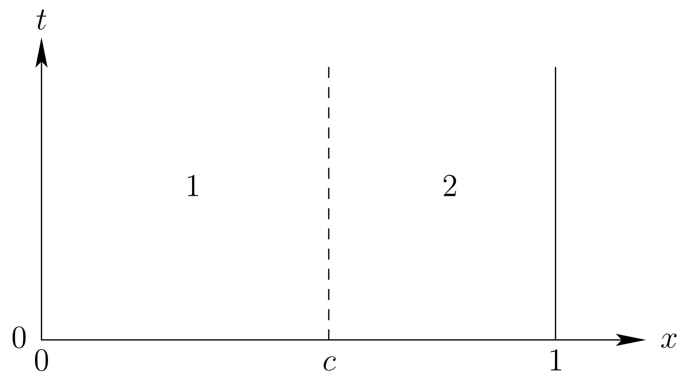
TMA4212 Numerical solution of differential equations using difference methods

Problem Set 3

Problem 1.



We have heat transfer in a plate composed of two materials. Find a method to solve the following problem:

$$
\begin{array}{ll}
u^1_t = a_1 u^1_{xx}, \quad 0 < x < c,\ t > 0, & u^1(x,0) = f_1(x),\ 0 \le x \le c \\
u^2_t = a_2 u^2_{xx}, \quad c < x < 1,\ t > 0, & u^2(x,0) = f_2(x),\ c < x \le 1 \\
u^1(0,t) = g_0(t),\ t > 0, & u^2(1,t) = g_1(t),\ t > 0 \\
u^1(c,t) = u^2(c,t),\ t > 0, & \lambda_1 u^1_x(c,t) = \lambda_2 u^2_x(c,t),\ t > 0
\end{array}
$$

Problem 2. Check if your formula from Problem 1 works by implementing it in MATLAB with $c = 1/2$ and

$$f_1(x) = 4x(1-x),\ f_2(x) = 4x(1-x),\ a_1 = 1,\ a_2 = 100,\ \lambda_1 = \lambda_2 = 1,\ g_0 \equiv g_1 \equiv 0$$

Problem 3. Let r be a positive real number. Show that if $U = (U_1, \dots, U_M)^T$ satisfies

$$-rU_{m-1} + (1+2r)U_m - rU_{m+1} = v_m, \quad 1 \le m \le M-1$$

$$U_0 = U_M = 0$$

then

$$\max_{0 \le m \le M} |U_m| \le \max_{1 \le m \le M-1} |v_m|$$

Use this to show that Backwards Euler converges for arbitrary $r = k/h^2$ on the problem

$$u_t = u_{xx},\ u(x,0) = f(x),\ u(0,t) = g_0(t),\ u(1,t) = g_1(t)$$

Problem 4. Find stability requirements for Euler's method on the problem

$$u_t = u_{xx} + \beta u \quad \text{and} \quad u_t = u_{xx} - \beta u, \quad \beta > 0$$

with the usual boundary conditions ($u(x,0),\ u(x,1),\ u(0,x)$ given).