# TMA4212 Numerical solution of partial differential equations with finite difference methods

**Problem Set 4**

**Problem 1.** A differential equation has a solution $u(x, y)$ on a rectangle in the $(x, y)$ plane. Suppose that this solution is approximated on a grid $(x_m, y_n)$, $x_m = mh$, $y_n = nk$ so that $U_{m,n} \approx u(x_m, y_n)$ satisfies the difference formula

$$aU_{m+1,n+1} + bU_{m-1,n+1} + cU_{m+1,n-1} + dU_{m-1,n-1} + eU_{m,n} = 0, \qquad 1 \le m \le M,\ 1 \le n \le N,$$

where $a, b, c, d, e$ are constants which may depend on the stepsizes $h, k$. The boundary values are $U_{m,n} = g(x_m, y_n)$ for $m = 0, m = M + 1, n = 0, n = N + 1$.

**a)** Draw the calculation molecule for this formula.

**b)** There are $MN$ unknowns. Define a column vector of unknowns, $U \in \mathbb{R}^{MN}$,

$$U = (U_{1,1}, \ldots, U_{M,1}, U_{1,2}, \ldots, U_{M,2}, \ldots, \ldots, U_{M,N})^T.$$

Find the matrix $A$ and vector $b$ so that the difference equations above can be expressed as

$$AU = b$$

**c)** If $M = N = 1000$, find an upper bound on the number of elements of $A$ that are nonzero.

**d)** Figure out how to generate $A$ in MATLAB. Avoid storing the zeros.
*Hint.* Investigate the functions `speye`, `spdiags` and `kron`.

**e)** We now work on the L-shaped domain shown in the figure. Suppose here that $M, N$ are odd numbers. Number the gridpoints line by line as before. Redo parts **a)**, **b)** and **c)**.

**f)** Find values of $a, b, c, d, e$ so that the difference equation approximates the solution of

$$u_{xx} + u_{yy} = 0.$$