TMA4212 Numerical solution of differential equations with difference methods

Problem Set 5

Problem 1. A difference equation has the form

$$AU^{n+1} = BU^n + d^n$$

and the truncation error τ^n is defined as

$$Au^{n+1} = Bu^n + d^n + \tau^n.$$

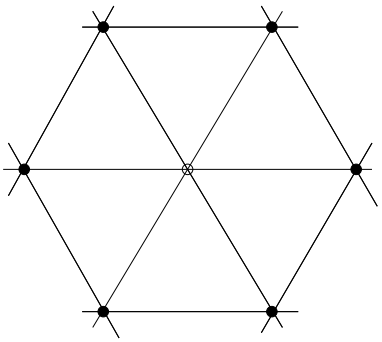
Show that the discretization error $e^n = u^n - U^n$ can be written

$$e^n = v^{n-1} + Cv^{n-2} + \cdots + C^{n-2}v^1 + C^{n-1}v^0$$

where $C = A^{-1}B$, $v^l = A^{-1}\tau^l$, and where we have assumed $e^0 = \mathbf{0}$. Assume that $\|C^n\| \leq L$ for all n and show that

$$\|e^n\| \leq T \cdot L \cdot \max_{l \leq T/k} \|\frac{1}{k} v^l\| \quad \text{for enhver } n \leq T/k.$$

Problem 2. Find a discretization of the Laplacian $\Delta = \partial_x^2 + \partial_y^2$ on a hexagonal grid (see figure). Give the first term of the truncation error. *Hing:* Use symmetry.



Problem 3. Find one or more difference approximations to $\partial_x \partial_y u$ on a uniform rectangular grid (same steplength in both directions)

Problem 4. Discretize $\Delta u = f$ i P with the condition $\partial_n u + au = g$ along the line γ (see figure) using box integration.