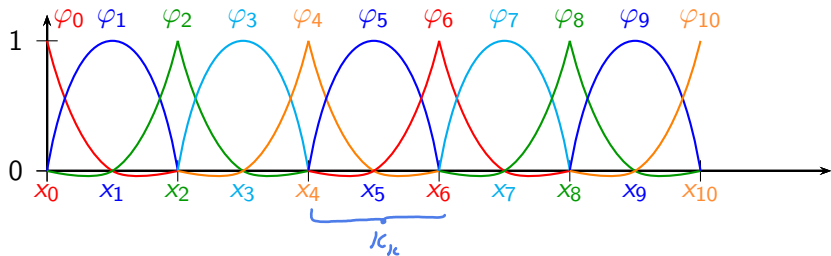


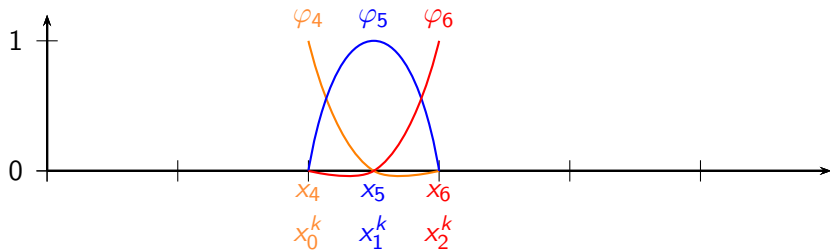
Basis functions:



Elements: $\kappa_k = (x_{2k-2}, x_{2k})$, $k=1, \dots, 5$

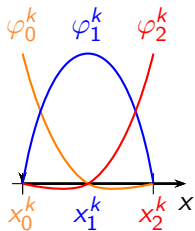
Element size: $h_k = x_{2k} - x_{2k-2}$

Pick out element $k = 3$

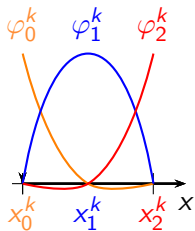


Local-to-global mapping: $\theta(k, \alpha) = 2(k - 1) + \alpha, \alpha = 0, 1, 2.$

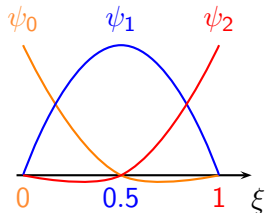
Element K_k :



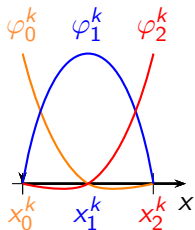
Element K_k :



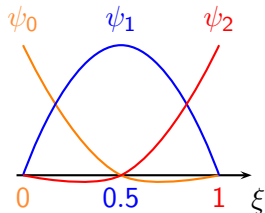
Reference element \hat{K} :



Element K_k :



Reference element \hat{K} :



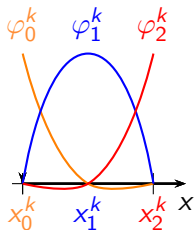
Shape functions:

$$\psi_0(\xi) = 2(\xi - \frac{1}{2})(\xi - 1)$$

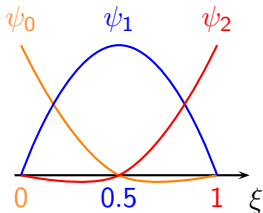
$$\psi_1(\xi) = -4\xi(\xi - 1)$$

$$\psi_2(\xi) = 2\xi(\xi - \frac{1}{2})$$

Element K_k :



Reference element \hat{K} :



Mapping from $\hat{K} \Leftrightarrow K_k$:

$$\varphi_\alpha^k(x) = \psi_\alpha(\xi(x)) = \psi_\alpha(\Phi_k^{-1}(x)), \quad \alpha = 0, 1, 2$$

$$x(\xi) = \Phi_k(\xi) = x_0^k + h_k \xi$$

$$\xi(x) = \Phi_k^{-1}(x) = (x - x_0^k)/h_k, \quad h_k = x_2^k - x_0^k$$

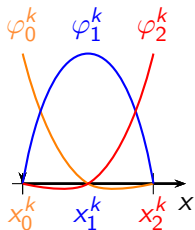
Shape functions:

$$\psi_0(\xi) = 2(\xi - \frac{1}{2})(\xi - 1)$$

$$\psi_1(\xi) = -4\xi(\xi - 1)$$

$$\psi_2(\xi) = 2\xi(\xi - \frac{1}{2})$$

Element K_k :



$$\begin{aligned}\tilde{A}_{h,\alpha,\beta}^k &= a(\varphi_\alpha^k, \varphi_\beta^k)|_{K_k} = \int_{x_0^k}^{x_2^k} \frac{d\varphi_\alpha^k}{dx} \frac{d\varphi_\beta^k}{dx} dx \\ &= \int_0^1 \left(\frac{d\Psi_\alpha}{d\xi} \frac{d\Phi^{-1}}{dx} \right) \left(\frac{d\Psi_\beta}{d\xi} \frac{d\Phi^{-1}}{dx} \right) \frac{d\Phi}{dx} d\xi \\ &= \frac{1}{h_k} \int_0^1 \frac{d\Psi_\alpha}{d\xi} \frac{d\Psi_\beta}{d\xi} d\xi\end{aligned}$$

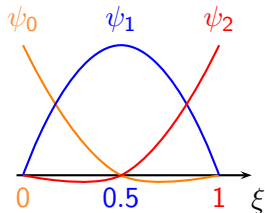
Mapping from $\hat{K} \Leftrightarrow K_k$:

$$\varphi_\alpha^k(x) = \psi_\alpha(\xi(x)) = \psi_\alpha(\Phi_k^{-1}(x)), \quad \alpha = 0, 1, 2$$

$$x(\xi) = \Phi_k(\xi) = x_0^k + h_k \xi$$

$$\xi(x) = \Phi_k^{-1}(x) = (x - x_0^k)/h_k, \quad h_k = x_2^k - x_0^k$$

Reference element \hat{K} :



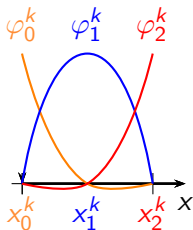
Shape functions:

$$\psi_0(\xi) = 2(\xi - \frac{1}{2})(\xi - 1)$$

$$\psi_1(\xi) = -4\xi(\xi - 1)$$

$$\psi_2(\xi) = 2\xi(\xi - \frac{1}{2})$$

Element K_k :

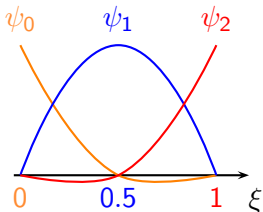


Element matrix \tilde{A}_h^k :

$$\tilde{A}_{h,\alpha,\beta}^k = \frac{1}{h_k} \int_0^1 \frac{d\Psi_\alpha}{d\xi} \frac{d\Psi_\beta}{d\xi} d\xi$$

$$\tilde{A}_h^k = \frac{1}{h_k} \begin{pmatrix} \frac{7}{3} & -\frac{8}{3} & \frac{1}{3} \\ -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} \\ \frac{1}{3} & -\frac{8}{3} & \frac{7}{3} \end{pmatrix}$$

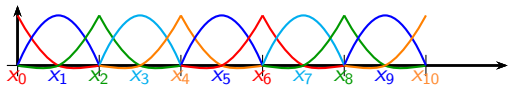
Reference element \hat{K} :



$$\psi_0(\xi) = 2(\xi - \frac{1}{2})(\xi - 1)$$

$$\psi_1(\xi) = -4\xi(\xi - 1)$$

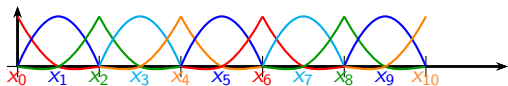
$$\psi_2(\xi) = 2\xi(\xi - \frac{1}{2})$$



$$\tilde{A}_h^k = \frac{1}{h_k} \begin{pmatrix} \frac{7}{3} & & \\ -\frac{8}{3} & & \\ \frac{1}{3} & & \\ & -\frac{8}{3} & \\ & \frac{16}{3} & \\ & & \frac{1}{3} \end{pmatrix}$$

$$i = \theta(k, \alpha) = 2(k - 1) + \alpha, \quad \alpha = 0, 1, 2, \quad h_k = 2h$$

$$\tilde{A}_h = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

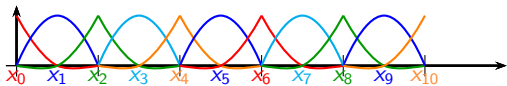


$$\tilde{A}_h^k = \frac{1}{h_k} \begin{pmatrix} \frac{7}{3} & -\frac{8}{3} & \frac{1}{3} \\ -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} \\ \frac{1}{3} & -\frac{8}{3} & \frac{7}{3} \end{pmatrix}$$

$$i = \theta(k, \alpha) = 2(k - 1) + \alpha, \quad \alpha = 0, 1, 2, \quad h_k = 2h$$

$k=1$

$$\tilde{A}_h = \frac{1}{2h} \begin{pmatrix} \frac{7}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & -\frac{8}{3} & \frac{7}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

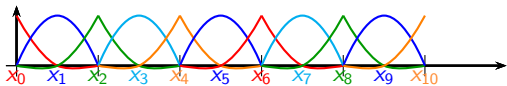


$$\tilde{A}_h^k = \frac{1}{h_k} \begin{pmatrix} \frac{7}{3} & -\frac{8}{3} & \frac{1}{3} \\ -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} \\ \frac{1}{3} & -\frac{8}{3} & \frac{14}{3} \\ 0 & 0 & -\frac{8}{3} \\ 0 & 0 & \frac{16}{3} \\ 0 & 0 & -\frac{8}{3} \\ 0 & 0 & \frac{7}{3} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$i = \theta(k, \alpha) = 2(k - 1) + \alpha, \quad \alpha = 0, 1, 2, \quad h_k = 2h$$

$$k=2$$

$$\tilde{A}_h = \frac{1}{2h} \begin{pmatrix} \frac{7}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & -\frac{8}{3} & \frac{14}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & -\frac{8}{3} & \frac{7}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$



$$\tilde{A}_h^k = \frac{1}{h_k} \begin{pmatrix} \frac{7}{3} & & & & & & & & & & \\ -\frac{8}{3} & \frac{16}{3} & & & & & & & & & \\ \frac{1}{3} & -\frac{8}{3} & \frac{14}{3} & & & & & & & & \\ & 0 & -\frac{8}{3} & \frac{16}{3} & & & & & & & \\ 0 & 0 & \frac{1}{3} & -\frac{8}{3} & \frac{14}{3} & & & & & & \\ 0 & 0 & 0 & 0 & -\frac{8}{3} & \frac{16}{3} & & & & & \\ 0 & 0 & 0 & 0 & \frac{1}{3} & -\frac{8}{3} & \frac{14}{3} & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{8}{3} & \frac{16}{3} & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & -\frac{8}{3} & \frac{14}{3} & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{8}{3} & \frac{16}{3} & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & -\frac{8}{3} & \frac{14}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{8}{3} & \frac{16}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & -\frac{8}{3} & \frac{14}{3} \end{pmatrix}$$

$$i = \theta(k, \alpha) = 2(k - 1) + \alpha, \quad \alpha = 0, 1, 2, \quad h_k = 2h$$

$$k=4$$

$$\tilde{A}_h = \frac{1}{2h} \begin{pmatrix} \frac{7}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & -\frac{8}{3} & \frac{14}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & -\frac{8}{3} & \frac{14}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & -\frac{8}{3} & \frac{14}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & -\frac{8}{3} & \frac{14}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & -\frac{8}{3} & \frac{14}{3} & -\frac{8}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & -\frac{8}{3} & \frac{14}{3} & -\frac{8}{3} \end{pmatrix}$$



$$\tilde{A}_h^k = \frac{1}{h_k} \begin{pmatrix} \frac{7}{3} & & & & & & & & & & & \\ & -\frac{8}{3} & & & & & & & & & & \\ & & \frac{16}{3} & & & & & & & & & \\ & & & -\frac{8}{3} & & & & & & & & \\ & & & & \frac{14}{3} & & & & & & & \\ & & & & & -\frac{8}{3} & & & & & & \\ & & & & & & \frac{16}{3} & & & & & \\ & & & & & & & -\frac{8}{3} & & & & \\ & & & & & & & & \frac{14}{3} & & & \\ & & & & & & & & & -\frac{8}{3} & & \\ & & & & & & & & & & \frac{16}{3} & \\ & & & & & & & & & & & -\frac{8}{3} \\ & & & & & & & & & & & & \frac{7}{3} \end{pmatrix}$$

$$i = \theta(k, \alpha) = 2(k - 1) + \alpha, \quad \alpha = 0, 1, 2, \quad h_k = 2h$$

$$k=5$$

$$\tilde{A}_h = \frac{1}{2h} \begin{pmatrix} \frac{7}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & -\frac{8}{3} & \frac{14}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & -\frac{8}{3} & \frac{14}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & -\frac{8}{3} & \frac{14}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & -\frac{8}{3} & \frac{14}{3} & -\frac{8}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & -\frac{8}{3} & \frac{14}{3} & -\frac{8}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & -\frac{8}{3} & \frac{7}{3} \end{pmatrix}$$

$$\tilde{\mathbf{A}}_h \mathbf{u} = \tilde{\mathbf{F}}_h$$

or

$$\frac{1}{2h} \begin{pmatrix} \frac{7}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & -\frac{8}{3} & \frac{14}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & -\frac{8}{3} & \frac{14}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & -\frac{8}{3} & \frac{14}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & -\frac{8}{3} & \frac{14}{3} & -\frac{8}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & -\frac{8}{3} & \frac{14}{3} & -\frac{8}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & -\frac{8}{3} & \frac{7}{3} \end{pmatrix} \begin{pmatrix} u_0 \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \\ u_9 \\ u_{10} \end{pmatrix} = \begin{pmatrix} F_0 \\ F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \\ F_7 \\ F_8 \\ F_9 \\ F_{10} \end{pmatrix}$$