

TMA4215 Numerical Mathematics

Autumn 2010

Exercise 0

No exercise session.

The intention of this exercise is to get you started using MATLAB. First, either find a PC with MATLAB installed, or install it on your own laptop. Start MATLAB. The first thing you should do is to spend some time familiarizing yourself with the user interface. The main windows you will be using is the command window and a text editor. MATLAB has its own built-in editor which we recommend that you use, however if you rather want to use one that you are familiar with, that is also fine. The main reason to use the built-in editor is that it has a debugger, which will certainly come in handy when you try to find bugs in your programs.

MATLAB is a powerful tool, with tons of functionality and features. The main things you should be able to handle are:

- Use of scalars, vectors and matrices and how to do basic operations on these.
- Flow-control, mainly `if`-statements and `for`-loops.
- Functions and scripts.
- Simple plots.
- And, most importantly, how to utilize MATLAB's help functions `help`, `doc` and `lookfor`.

We have a very good tutorial written by Håvard Berland (see <http://wiki.math.ntnu.no/drift/stud/matlab>). Unfortunately, this is only available in Norwegian. However, there are many good tutorials available at http://www.mathworks.com/academia/student_center/tutorials/launchpad.html. For instance the one from Michigan Tech seems to be nice.

Preparations: Read the tutorial of your choice, and test the things you learn there. When you feel ready, you can move on to the rest of the tasks.

Task 1

Given

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix}, \quad b = \begin{bmatrix} 7 \\ 7 \\ 6 \end{bmatrix}.$$

- Solve the linear system of equations $Ax = b$. Check that the answer is correct.
- Find the eigenvalues and eigenvectors of A . You have to find the correct command and how to use it yourself.

Task 2

In this task you are asked to develop your own little MATLAB program. If you are a programming novice, my advice is: *Think slowly!* Do not write a single line of code before you are certain you know what the program is supposed to do, and how it should do it. If you think doing the task by hand or calculator first would help, that is what you should do.

In introductory calculus you learned Newton's method for finding the roots of nonlinear equations, i.e. how to solve $f(x) = 0$ for a nonlinear $f(x)$. The method is given by

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, \dots$$

- a) Write a script where you use Newton's method to find $\sqrt{3}$, that is to solve the equation $x^2 - 3 = 0$. Use $x_0 = 2$ as the initial guess and do 5 iterations.
- b) Add a stopping criterion. Here, this means that the program should keep iterating until the difference between two iterates is less than a given error bound, for instance

$$|x_{n+1} - x_n| < 10^{-6}.$$

Task 3

Here we want you to modify the program `euler.m`, which is a small script that solves an ordinary differential equation using Euler's method,

$$y_{n+1} = y_n + hf(t_n, y_n).$$

Modify the script in order to make it solve the equation

$$\begin{aligned} u' &= u(1 - v), & u(0) &= 1, \\ v' &= v(u - 2), & v(0) &= 2, \end{aligned}$$

for $0 \leq t \leq 10$. Use $h = 0.1$. Plot the obtained results.

Hint: Write a function of the form `function yp = f(t, y)` where `yp` and `y` are 2-dimensional column vectors, and `t` is a scalar. The name of this function must be given as input to `euler.m`.