TMA4215 Numerical Mathematics

Autumn 2010

Exercise 1

You should have read 3.2 and 3.3 in K&C, in addition to the section about Newton's method in the note on nonlinear equations before solving these exercises.

Task 1

a) Apply Newton's method to the equation $f(x) = 0$ where

i) $f(x) = \cos x - 1/2$, with $x_0 = 0.5$.
ii) $f(x) = \mathrm{e}^x - x - 1$, with $x_0 = 0.5$.
iii) $f(x) = x(1 - \cos x)$, with $x_0 = 0.5$.

The iterations will converge to a zero $x^\star$ in all three cases. Measure the order of convergence using e.g. MATLAB in the three cases. Is the result in accordance with theory? If no, can you explain why?

We say that a zero $x^\star$ of $f(x)$ has multiplicity m if there exists a function $q(x)$ such that

$$f(x) = (x - x^\star)^m q(x), \qquad q(x^\star) \neq 0,$$

which is the case if and only if

$$f(x^\star) = f'(x^\star) = \cdots = f^{(m-1)}(x^\star) = 0, \qquad f^{(m)}(x^\star) \neq 0.$$

b) What is the multiplicity of the solutions of the three equations in **a)**?

c) Assume that $x^\star$ is a zero with multiplicity m of the function $f(x)$. Show that the function

$$\mu(x) = f(x)/f'(x)$$

has a simple zero in $x^\star$, independent of m. Use this to find an iteration scheme that converges quadratically to $x^\star$.

d) Test the new scheme on the functions *ii)* and *iii)* in **a)**.

e) Repeat the task using the secant method instead of Newton's method.

Task 2

Consider the system of equations

$$x_1^2 + x_2^2 = 1,$$
$$x_1^3 - x_2 = 0.$$

This has two solutions, one in the region $-1 \leq x_1, x_2 \leq 0$ and one in $0 \leq x_1, x_2 \leq 1$.

a) Choose appropriate initial values and perform two iterations by hand using Newton's method.

b) Verify that you get correct answers using MATLAB.

c) Explain what happens when you choose initial value lying on the x_1-axis.