Norwegian University of Science and Technology
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TMA4215 Numerical mathematics
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Solutions to exercise set 10

1 Set 7, Problem 3, 4 and 5: See the solution set

2 **a)** See section 7.1 in the Lecture note on numerical solution of ordinary differential equations.

We have 4 free parameter, so we search for a method of order 3, that is we try to solve $C_0 = C_1 = C_2 = C_3 = 0$, or

$$\begin{aligned}
C_0 &= 1 - \alpha_1 + a = 0 \\
C_1 &= 2 - \alpha_1 - \beta_2 - \beta_1 - \beta_0 = 0 \\
C_2 &= \frac{1}{2}(4 - \alpha_1) - 2\beta_2 - \beta_1 = 0 \\
C_3 &= \frac{1}{6}(8 - \alpha_1) - \frac{1}{2}(4\beta_2 + \beta_1) = 0.
\end{aligned}$$

with the solution

$$\alpha_1 = 1 + a, \quad \beta_0 = -\frac{1+5a}{12}, \quad \beta_1 = \frac{2-2a}{3}, \qquad \beta_2 = \frac{5+a}{12}.$$

The error constant is

$$C_4 = -\frac{1+a}{24}.$$

b) Clearly, the method is consistent ($C_0 = C_1 = 0$). We need to check zero-stability. The characteristic polynomial is

$$\rho(r) = r^2 - (1+a)r + a$$

with roots $r_1 = 1$ and $r_2 = a$. So the method is zero-stable, and thereby convergent, for $-1 \le a < 1$.

Notice: For $a = -1$ the method is of order 4, and convergent.

If $a = 1$, we get the third order Adams-Moulton method (see Example 7.10).