NTNU – Trondheim
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Department of Mathematical Sciences

Examination paper for **TMA4215 Numerical Mathematics**

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Examination time (from–to): 09:00–13:00

Permitted examination support material: C:

- Endre Süli and David Mayers *An Introduction to Numerical Analysis* (a printout/copy is accepted)
- *TMA4215 Numerical Mathematics: Collection of Lecture Notes* (15. November 2013, 62 pages)
- Rottmann: *Matematisk formelsamling*
- Approved calculator

Language: English

Number of pages: 3

Number pages enclosed: 0

Checked by:

Date Signature

Problem 1

a) Is the following matrix symmetric positive definite?

$$\begin{pmatrix} 4.5 & -1.0 & -2.0 \\ -1.0 & 3.5 & 2.3 \\ -2.0 & 2.3 & 5.6 \end{pmatrix} \tag{1}$$

b) What matrix quantity is important in order to characterize the expected accuracy when numerically solving a linear equation system involving a symmetric positive definite matrix?

Problem 2

a) The value of a function $f(x)$ is known in the points:

x_i	0.0	2.0	3.0
$f(x_i)$	2.0	12.0	20.0

Find the polynomial $p(x)$ of lowest possible order interpolating $f(x)$ in these three points.

b) Find the polynomial $q(x)$ of lowest possible order which satisfies the interpolation conditions above *in addition* to the condition $f(1.0) = 6.0$.

c) Construct the Hermite interpolation polynomial of degree 3 for the function $f(x) = x^5$, using the points $x_0 = 0$, $x_1 = a$, and show that it has the form $p_3(x) = 3a^2x^3 - 2a^3x^2$.

d) Verify Theorem 6.4 on page 190 in the textbook by Süli and Mayers by direct calculation, showing that in this case ξ is unique and has the value $\xi = \frac{1}{5}(x + 2a)$.

Problem 3

a) Determine the values c_j, $j = -1, 0, 1, 2$, such that the quadrature rule $Q(f) = c_{-1}f(-1) + c_0 f(0) + c_1 f(1) + c_2 f(2)$ gives the correct value for the integral $\int_0^1 f(x)dx$ when f is any polynomial of degree 3.

b) Define the composite trapezium rule T_m, the composite Simpson rule S_m and the composite midpoint rule M_m, each with m subintervals. Show that: $M_m = 2T_{2m} - T_m$, $S_m = (4T_{2m} - T_m)/3$, and $S_m = (2M_m + T_m)/3$

c) Which numerical integration methods would you consider using for solving the following integral?

$$\int_{-5}^{5} \frac{1}{1+x^2}\, dx \tag{2}$$

d) Discuss the characteristics of orthogonal polynomials and the importance that play in approximation of functions and numerical integration. Construct one set of orthogonal polynomials (including polynomials of order 0, 1 and 2). Specify the orthogonality condition and verify that your set of polynomials form an orthogonal set.

Problem 4 Given a scalar ordinary differential equation (ODE)

$$y' = f(t, y), \quad t_0 \le t \le t_{\text{end}}, \quad y(t_0) = y_0 \tag{3}$$

in which the function f, the integration interval $[t, t_{\text{end}}]$ and the initial value y_0 is assumed to be given.

a) Show that the one-step method defined by

$$y_{n+1} = y_n + \frac{1}{2}h(k_1 + k_2) \tag{4}$$

where

$$k_1 = f(x_n, y_n), \quad k_2 = f(x_n + h, y_n + hk_1) \tag{5}$$

is consistent. Find the local truncatione error for this method.

b) What is the stability region for the one-step method given above? Is it absolute stable (A-stable)?

c) Find the order of the following linear multistep method:

$$y_{n+2} + 4y_{n+1} - 5y_n = h(2f_n + 4f_{n+1}) \tag{6}$$

d) Is the linear multistep method above convergent?